The Procedure for Coding the Gaussian Elimination / Backward Substitution Method for Solving Large (1000's) of Nodal and other sets of equations

Gaussian Elimination Method for Solving a set of Equations. Consider two equations:

$$\#1 \quad Ax + By + C = 0$$

$$\#2 \quad Dx + Ey + F = 0$$

A, B, C, D, E, F Are known.

$\boxed{x \text{ and } y \text{ are unknown}}$

Since both equations $= 0$, you can add or subtract the equations or multiples of the equations without affecting the final solutions for x and y.

Gaussian Elimination says to subtract a multiple of #1 from #2 so that the "modified" D becomes zero.

If we multiply #2 by $\frac{A}{D}$ we get

$$\left(\frac{A}{D}\right)\cdot Dx + \left(\frac{A}{D}\right)\cdot Ey + \left(\frac{A}{D}\right)\cdot F = 0$$

yielding mod. #2 $\left[Ax + \frac{AE}{D}y + \frac{AF}{D} = 0\right]$

The new pair of equations is

$$1 \quad Ax + By + C = 0$$

$$2'' \text{mod} \quad Ax + \frac{AE}{D}y + \frac{AF}{D} = 0$$

Now subtract 1 from 2'', yielding

$$1 \quad Ax + By + C = 0$$

$$2 \text{ mod} \quad 0 + \left(\frac{AE}{D} - B\right)y + \left(\frac{AF}{D} - C\right) = 0$$

Thus, $y = \left[\dfrac{C - \frac{AF}{D}}{\frac{AE}{D} - B}\right]$

For 3x3,

$$\begin{cases} Ax + By + Cz + J = 0 \\ Dx + Ey + Fz + K = 0 & \Leftarrow \text{mult. by } \frac{A}{D} \text{ to eliminate D.} \\ Gx + Hy + Iz + L = 0 & \leftarrow \text{mult. by } \frac{A}{G} \text{ to eliminate G.} \end{cases}$$

Creating modified equations

$$\begin{array}{llllll} \#1 & \boxed{Ax + By + Cz + J = 0} \\ \#2' & 0 + \boxed{E'y + F'z + K' = 0} \\ \#3' & 0 \quad \boxed{H'y + I'z + L' = 0} \end{array}$$

Now, repeat the process for the two equations

$$\begin{array}{l} \#2' \\ \#3' \end{array} \begin{bmatrix} E'y + F'z + K' = 0 \\ H'y + I'z + L' = 0 \end{bmatrix}$$

And eliminate $H'y$, so

$$\begin{array}{l} \#2' \\ \#3'' \end{array} \left[\begin{array}{l} E'y + F'z + K' = 0 \\ 0 \quad \boxed{I''z + L'' = 0} \end{array}\right]$$

Now, $I''z = -L''$

So, from #3'' $\boxed{z = \frac{-L''}{I''}}$!

Now, use backward substitution

Insert known z into #2' to compute y.

The insert known z and y into #1 to compute x.

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Your Problem

① $1x_1 - 2x_2 - 3x_3 - 4 = 0$

② $5x_1 + 6x_2 + 7x_3 - 8 = 0$

③ $9x_1 + 10x_2 + 11x_3 - 12 = 0$

② is now $\frac{1}{5}[5x_1 + 6x_2 + 7x_3 - 8] = 0$

$\hookrightarrow x_1 + \frac{6}{5}x_2 + \frac{7}{5}x_3 - \frac{8}{5} = 0$

③ is now $\frac{1}{9}[9x_1 + 10x_2 + 11x_3 - 12] = 0$

$\hookrightarrow x_1 + \frac{10}{9}x_2 + \frac{11}{9}x_3 - \frac{12}{9} = 0$

① $x_1 \quad -2x_2 \quad -3x_3 \quad -4 \quad = 0$

② $x_1 \quad +\frac{6}{5}x_2 \quad +\frac{7}{5}x_3 \quad -\frac{8}{5} \quad = 0$

③ $x_1 \quad +\frac{10}{9}x_2 \quad +\frac{11}{9}x_3 \quad -\frac{12}{9} \quad = 0$

②'' = ② − ① $\Rightarrow 0x_1 + \left(\frac{6}{5}+2\right)x_2 + \left(\frac{7}{5}+3\right)x_3 - \frac{8}{5} + 4 = 0$

③'' = ③ − ① $\Rightarrow 0x_1 + \left(\frac{10}{9}+2\right)x_2 + \left(\frac{11}{9}+3\right)x_3 - \frac{12}{9} + 4 = 0$

Simplifying $2'' \Rightarrow \left(\frac{16}{5}\right)x_2 + \left(\frac{22}{5}\right)x_3 + \left(\frac{12}{5}\right) = 0$

$3'' \Rightarrow \left(\frac{28}{9}\right)x_2 + \left(\frac{38}{9}\right)x_3 + \left(\frac{24}{9}\right) = 0$

Multiply 2'' by 5, Multiply 3'' by 9, $\begin{bmatrix} 16x_2 + 22x_3 + 12 = 0 \\ 28x_2 + 38x_3 + 24 = 0 \end{bmatrix}$

Eliminate x_2 by Multiplying 3'' by $\frac{16}{28}$

2'' $16x_2 + 22x_3 + 12 = 0$

3'' $\frac{16}{28}(28)x_2 + \frac{16}{28}(38)x_3 + \frac{16}{28}(24) = 0$

$3''' = 3'' - 2'' \Rightarrow 0x_2 + \left(\frac{16}{28}(38) - 22\right)x_3 + \frac{16(24)}{28} - 12 = 0$

mult. by 28, $[(16)(38) - 616]x_3 + 384 - 336 = 0$

$-8x_3 = -48$

$\boxed{x_3 = 6}$

Sub into 2'', $16x_2 + 22x_3 + 12 = 0$

$16x_2 + 22(6) + 12 = 0$

$\boxed{x_2 = -9}$

So, why do All this - so you
can see that given the
procedure, you could
write the computer!
code to do it

NOW

LAST Step

$x_1 = 2x_2 + 3x_3 + 4$

$= 2(-9) + 3(6) + 4$

$= -18 + 18 + 4$, $\boxed{x_1 = 4}$

Check

② $5(4) + 6(-9) + 7(6) - 8 = 0$ Yeah!

Check ③

$9x_1 + 10x_2 + 11x_3 - 12$

$9(4) + 10(-9) + 11(6) - 12 = 0$

Yeah!!