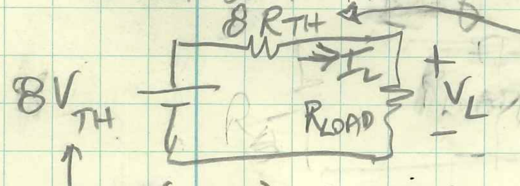
2021_0830

HW #1, cont.

Power is I^2R and VI. V and I are linear, but their product (Power) is non linear.
I.E., double the voltage across a resistor, and the power increases by a factor of 4. Nevertheless, we can attempt to find the R_{LOAD} that will consume 10 W.

Stick with the equations!

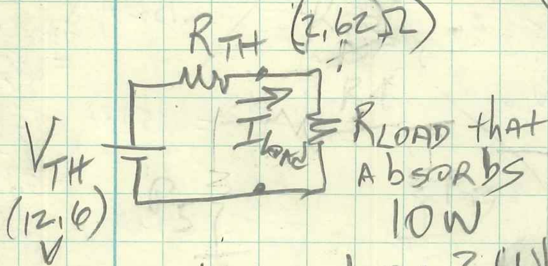


$8(0.328) = 2.62\,\Omega$

$8(1.575) = 12.6\text{V}$

What R_{LOAD} results in $P_L = 10\text{ W}$?

$$\underbrace{V_{LOAD} I_{LOAD} = 10\text{ W}, \quad \frac{V_{LOAD}^2}{R_{LOAD}} = 10\text{ W}, \quad I_{LOAD}^2 R_{LOAD} = 10\text{ W}}_{\text{One Equation shown three ways}}$$



Can this battery do it? What is it's max power load?

$$P_{LOAD} = V_{LOAD} I_{LOAD} = (\underbrace{V_{TH} - I_{LOAD} R_{TH}}_{V_{LOAD}})(I_{LOAD})$$

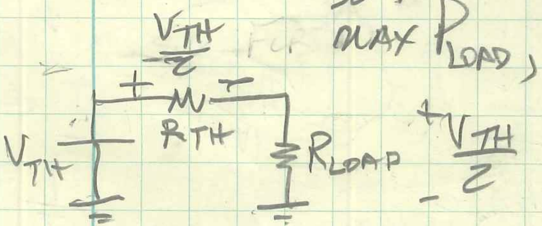
$$P_{LOAD} = \underbrace{V_{TH} I_{LOAD}}_{\substack{\text{Power}\\\text{out of}\\\text{battery}}} - \underbrace{R_{TH} I_{LOAD}^2}_{\substack{\text{Power}\\\text{consumed}\\\text{by } R_{TH}}}$$

Power consumed by R_{LOAD} $= P_{LOAD}$

Find the I_{LOAD} that results in max P_{LOAD}

$$\frac{\partial P_{LOAD}}{\partial I_{LOAD}} = V_{TH} - 2R_{TH} I_{LOAD} = 0$$

So for max P_{LOAD}, $I_{LOAD} = \frac{V_{TH}}{2R_{TH}}$ so $[R_{TH} + R_{TH}]$ yields max load power



So $R_{LOAD} = R_{TH}$ yields max P_{LOAD}

For max load power, Internal Losses equals load power, 50% Efficient (bad)
Load voltage is $\frac{1}{2} V_{TH}$ (bad)

Our case, $P_{MAX\,LOAD} = \underbrace{\left[\frac{1}{2} V_{TH}\right]}_{\text{LOAD VOLTAGE}} \underbrace{\left[\frac{V_{TH}}{2R_{TH}}\right]}_{\text{LOAD CURRENT}} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(12.6)^2}{4(2.62)} = 15.15\text{W}$

HW #1, cont. So, 10W output is possible.

Refer back to $P_{LOAD} = (V_{TH} - I_{LOAD} R_{TH})(I_{LOAD})$

$$P_{LOAD} = V_{TH} I_{LOAD} - R_{TH} (I_{LOAD})^2, \text{ so}$$

$$P_{LOAD} - V_{TH} I_{LOAD} + R_{TH} (I_{LOAD})^2 = 0$$

$$(I_{LOAD})^2 - \frac{V_{TH}}{R_{TH}}(I_{LOAD}) + \frac{P_{LOAD}}{R_{TH}} = 0 \quad \text{quadratic}$$

$$I_{LOAD} = \frac{\frac{V_{TH}}{R_{TH}} \pm \sqrt{\left(\frac{V_{TH}}{R_{TH}}\right)^2 - 4\frac{P_{LOAD}}{R_{TH}}}}{2} \quad \text{Two possible values}$$

$$I_{LOAD} = \frac{1}{2}\left[4.81 \pm \sqrt{(4.81)^2 - 4\left(\frac{10}{2.62}\right)}\right]$$

$$= \frac{1}{2}\left[4.81 \pm \sqrt{23.1 - 15.27}\right] = \frac{4.81 \pm 2.79}{2}$$

= 3.8A and 1.01 Amps both work but!

High Amps, Low Eff. — Low Amps, Higher Eff

INEFFICIANT
For the 3.8 A case, $V_{LOAD} = 12.6 - (3.8)(2.62) = 2.64\text{ V}$

$$R_{LOAD} = \frac{2.64\text{V}}{3.8\text{A}} = 0.697\ \Omega$$

$$P_{LOAD} = \frac{(2.64\text{V})^2}{0.697\ \Omega} = 10.0\text{ W}\ !$$

EFFICIANT
For the 1.01 A case

$$V_{LOAD} = 12.6 - (1.01)(2.62) = 9.95\text{ V}$$

$$R_{LOAD} = \frac{9.95\text{V}}{1.01\text{A}} = 9.85\ \Omega$$

$$P_{LOAD} = \frac{(9.95)^2}{9.85} = 10.1\text{ W}\ !$$