Capacitors & Inductors

\[ Q = CV, \text{ capacitance } C, \text{ in Farads } F \]

\[ Q = \frac{C}{d} \]

\[ C = \frac{F}{\text{permittivity}} \]

\[ \text{permittivity} = \varepsilon_0 \]

\[ \text{Relative permittivity} = \varepsilon \]

\[ \text{Free space} \]

\[ \text{or air} \]

Two parallel plates, \( C = \frac{\varepsilon A}{d} \)

\( E_r = 1 \) for air, free space.

Area \( A \) (each plate), separated by distance \( d \)

\[ C = \frac{2\pi \varepsilon_0}{\ln \frac{b}{a}} \frac{F}{\text{m}} \]

\[ \lambda = C \frac{dV}{dt}, \quad V = \frac{1}{C} \int i \, dt \]

\( \lambda \) in \( \text{Coulombs} \)

Start with an uncharged cap, charge up,

\[ P(t) = V(t)I(t) = (V)(C \frac{dV}{dt}) \]

\[ W = \int_{0}^{t} P \, dt = \int_{0}^{t} CV \frac{dV}{dt} \, dt = C \int_{0}^{V} \frac{dV}{\lambda} = \frac{1}{2} C \lambda^2 \]

Compute \( Q, W \) for 50 V, 1600 \( \mu \)F cap and 250 V, 1600 \( \mu \)F cap

Caps in parallel, more area, so more net \( C \)

\[ \lambda = \lambda_1 + \lambda_2 + \lambda_3 = \lambda \]

Same \( V \), \( \lambda_1 + \lambda_2 + \lambda_3 = \lambda \)
\[
\dot{V} = \frac{\text{d}V}{\text{d}t} = \frac{1}{C_{\text{tot}}} \left( V_1 + V_2 + V_3 \right) = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \frac{\text{d}V}{\text{d}t}
\]

\[
C_{\text{tot}} = C_1 + C_2 + C_3 \quad \text{(Add in parallel)}
\]

\[
\text{(Parallel caps add like series resistors)}
\]

Series caps, net gap increases \(\rightarrow\) smaller \(C\)

\[
\begin{align*}
\dot{I} &= \frac{\text{d}I}{\text{d}t} = \frac{1}{C_1} \frac{\text{d}V_1}{\text{d}t} = \frac{1}{C_2} \frac{\text{d}V_2}{\text{d}t} = \frac{1}{C_3} \frac{\text{d}V_3}{\text{d}t} \\
V_1 &= \frac{1}{C_1} \int \text{d}t \\
V_2 &= \frac{1}{C_2} \int \text{d}t \\
V_3 &= \frac{1}{C_3} \int \text{d}t
\end{align*}
\]

\[
V_{\text{tot}} = V_1 + V_2 + V_3 = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \int \text{d}t
\]

From \(\dot{I} = \frac{\text{d}V_{\text{tot}}}{\text{d}t}\), we have \(V_{\text{tot}} = \frac{I}{C_{\text{tot}}} \int \text{d}t\)

So \(\frac{I}{C_{\text{tot}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\) (Series caps combine like parallel \(R\)'s)

Inductors \(\dot{I} = \frac{\text{d}I}{\text{d}t}\)

\[
V_L = L \frac{\text{d}I}{\text{d}t}
\]

\(L = \frac{\mu N^2 A}{d}\)

\[
\dot{I} = \frac{1}{L} \int \text{d}V_L
\]

Start with \(\dot{I}_L = 0\), \(P = V_L I_L = I_L \frac{\text{d}V_L}{\text{d}t}\), \(W = \int_0^T \frac{\text{d}W}{\text{d}t} \text{d}t\)

\[
W = L \int_0^T \text{d}V_L = \frac{1}{2} L I^2 \quad \text{Joules}
\]

Work out \(W\) for \(10A, 10\mu H\)
Series Inductors

\[ V_{\text{Tot}} = U_1 + U_2 + U_3 = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} \]

\[ V_{\text{Tot}} = L_{\text{Tot}} \frac{di}{dt} \quad \Rightarrow \quad U_{\text{Tot}} = (L_1 + L_2 + L_3) \frac{di}{dt} \]

Add like series R's

Parallel Inductors

\[ i_{\text{Tot}} = \frac{1}{L_1} \int U_{\text{TD}} + \frac{1}{L_2} \int U_{\text{TD}} + \frac{1}{L_3} \int U_{\text{TD}} \]

\[ = \frac{1}{L_{\text{Tot}}} \int U_{\text{TD}} \quad \text{see that} \]

\[ \frac{1}{L_{\text{Tot}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \]

Parallel L's combine like parallel R's

Summary

Capacitors

\[ i = C \frac{dV}{dt}, \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{Parallel} \]

\[ \frac{1}{C_{\text{Series}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{Series} \]

\[ W = \frac{1}{2} CV^2 \]

Voltage cannot change instantly (that would require \( \infty \) power) and \( \infty \) current spike

Inductors

\[ V = L \frac{di}{dt}, \quad i = \frac{1}{L} \int U_{\text{TD}} \]

\[ L_{\text{Series}} = L_1 + L_2 + \ldots \]

\[ L_{\text{Parallel}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \]

\[ W = \frac{1}{2} L I^2 \]

Current cannot change instantly (would require \( \infty \) power), \( \infty \) voltage spike
100 \mu F, \quad \frac{1}{2} C V^2 = \frac{1}{2} (1800 \times 10^{-3})(50)^2 = 22.5 J

33 \mu F, \quad \frac{1}{2} (33 \times 10^{-6})(50)^2 = 0.0413 J

1 \text{ kWh} = 1000(60)(60) = 3,600,000 = 3.6 \text{ MJ}
= 3412 \text{ BTU}

1 \text{ gal gasoline} = 127,576 \text{ BTU} = 36.3 \text{ kWh} = 130.88 \text{ MJ}

1 \text{ gal} = 768 \text{ teaspoons}
1 \text{ teaspoon of gas} = \frac{130.88 \text{ MJ}}{768} = 0.1704 \text{ MJ}

1 \text{ drop of gas} = 170408 \text{ J}

(1000 \text{ mAh}) (1.2 \text{ V}) = 1.2 \text{ Wh} = (1.2)(3600) = 4320 \text{ J}
ENERGIZER E92

Specifications

Classification: Alkaline
Chemical System: Zinc-Manganese Dioxide (Zn/MnO₂)
No added mercury or cadmium
Designation: ANSI-24A, IEC-LR03
Nominal Voltage: 1.5 volts
Nominal IR: 150 to 300 milliohms (fresh)*
Operating Temp: -18°C to 55°C (0°F to 130°F)
Typical Weight: 11.5 grams (0.4 oz.)
Typical Volume: 3.8 cubic centimeters (0.2 cubic inch)
Jacket: Plastic Label
Shelf Life: 7 years at 21°C (80% of initial capacity)
Terminal: Flat Contact

* For additional information, please reference the IR technical white paper.

Millamp-Hours Capacity
Continuous discharge to 0.8 volts at 21°C

Device Selection Guide:

Battery Selection Indicator

Photoflash
Games, Digital Audio
Lighting
Remote Control
Radio

High Drain Devices
Moderate Drain Devices
Low Drain Devices

Important Notice
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Enegizer E92

**Constant Power Performance**
Typical Characteristics (21°C)

- Graph showing service hours vs. discharge in milliwatts for voltages 0.9 and 1.2 volts.

**Constant Current Performance**
Typical Characteristics (21°C)

- Graph showing service hours vs. discharge in milliamperes for voltages 0.8, 1.0, and 1.2 volts.

**Constant Power Performance**
Discharge Characteristics (21°C)

- Graph showing voltage (CCV) vs. service hours for a 100 mW and 250 mW discharge.

**Constant Current Performance**
250 mA Discharge (-20°C / 0°C / 21°C)

- Graph showing voltage (CCV) vs. service hours for different temperatures.

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**Industry Standard Tests** (21°C)

**LIGHTING**
5.1 ohm LIF

- Graph showing voltage (CCV) vs. service hours for a 5.1 ohm load.

**REMOTE**
24 ohm 15 sec/min 8 hrs/day

- Graph showing voltage (CCV) vs. service hours for a remote device.

**TAPE-GAME-DIGITAL AUDIO**
100 mA 1 hr/day

- Graph showing voltage (CCV) vs. service hours for a tape-game-digital audio application.

**PHOTOFLASH**
600 mA 10 sec/min 1 hr/day

- Graph showing voltage (CCV) vs. service hours for a photoflash application.

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First Order Circuit, an ordinary first order differential equation
No mix of L's & C's. If multiple L's, they are combinable into one L_{tot}. Ditto for C's.

Consider \[ V = \frac{R}{t=0} \]

KVL, \[-V + Ri + \frac{1}{C} \int i dt = 0\]
\[
\frac{d}{dt} \Rightarrow R \frac{di}{dt} + \frac{i}{C} = 0
\]
\[
\frac{di}{dt} = -\frac{i}{RC}, \text{ try } i = i_0 e^{-t/RC}
\]

Sub in, \[-\frac{i_0}{RC} e^{-t/RC} = -\frac{i_0}{RC} e^{-t/RC} \quad \text{(OK)}\]

What is \( i_0 \)? \( t = 0 \)

\[ V \]

Let \( \tau \) seconds = \( RC \), the time constant \((\text{Big } C, \text{ long } \tau) \)

\[
\begin{align*}
\tau &= RC, e^{-1} = 0.368 \\
2 \tau &= e^{-2} = 0.1353 \\
3 \tau &= e^{-3} = 0.0498 \quad \text{(About 5\% of starting value)} \\
5 \tau &= e^{-5} = 0.0067 \quad \text{(Close to 1\% of starting value)} \\
10 \tau &= e^{-10} = 0.000045 \quad \text{(Practically zero)}
\end{align*}
\]

What about \( V_c \)? Use \( V_c = \frac{1}{C} \int i dt = \frac{1}{C} \int i_0 e^{-t/RC} dt \)

\[
V_c = -\frac{i_0}{RC} e^{-t/RC} + V_{co} = -\frac{i_0}{RC} (e^{-t/RC} - 1) + V_{co}
\]

\[
V_c(t) = V - (V - V_{co}) e^{-t/RC} = V_F + (V_I - V_F) e^{-t/RC}
\]

Here, \( V_F = V \), \( V_I = V_{co} \)}


\[ v_c(t) = V_F + (V_I - V_F) e^{-t/\tau} \]

\( t = 0, \quad v_c(0) = V_F + (V_I - V_F) e^{-0} = V_F + V_I - V_F = V_I \quad \text{(OK)} \)

\( t \to \infty, \quad v_c(\infty) = V_F \quad \text{(OK)} \)

You'll use this form many times. And, for a circuit with \( V' \)'s and \( R' \)'s, use

\[ \frac{V_{TH}}{C} \quad \tau = \frac{R_{TH}}{C} \quad V_F = V_{TH} \]

All voltages and currents in the circuit will have the form

\[ V_F + (V_I - V_F) e^{-t/\tau} \quad \text{same } \tau, \text{ different } V_F, V_I \]

\[ I_F + (i_I - I_F) e^{-t/\tau} \quad \text{ same } \tau, \text{ different } I_F, i_I \]

So, the procedure is find \( V_{TH}, R_{TH}, V_F, V_I \).

---

For \( L' \)'s, same procedure, different \( \tau \)

\[ \frac{R_{TH}}{L} \quad \text{(of course, in this picture, } \bar{i}(0^-) = 0) \]

\[ -V_{TH} + R_{TH} \bar{i} + L \frac{d\bar{i}}{dt} = 0. \quad \text{Lets try to get an } S \text{ instead} \]

\[ \frac{-V_{TH}}{R_{TH}} + \bar{i} + \frac{L}{R_{TH}} \frac{di}{dt} = 0 \]

\[ \frac{d}{dt} \int V_L dt + \frac{V_L - V_{TH}}{R_{TH}} = 0 \]

\[ \frac{d}{dt} \int V_L dt + \frac{V_L - V_{TH}}{R_{TH}} = 0 \]

\[ \frac{dV_L}{dt} = -V_L \frac{R_{TH}}{L} \]

(Guess \( V_L = V_{LO} e^{-t/\tau_1} \))

\[ -V_{LO} \frac{R_{TH}}{L} e^{-t/\tau_1} = -V_{LO} \frac{R_{TH}}{L} e^{-t/\tau_1} \]

\[ \text{so } \tau = \frac{R_{TH}}{L} \]
What about \( i(t) \)?

\[
i(t) = \frac{V_{TH} - V_L(t)}{R_{TH}} = \frac{V_{TH}}{R_{TH}} - \frac{V_{TH}}{R_{TH}} e^{-t/\tau} = \frac{V_{TH}}{R_{TH}} (1 - e^{-t/\tau})
\]

So if no initial current, \( L \) starts as open ckt, and ends as a short ckt.

The process ends up with the same form,

\[
V_F + (V_I - V_F) e^{-t/\tau} \quad \Rightarrow \quad \frac{V_F + (V_I - V_F)}{R_{TH}} e^{-t/\tau}
\]

\[
\tau = \frac{L}{R_{TH}}
\]

Big \( L \), long \( \tau \), small \( R \), long \( \tau \).

The slope of \( V_F + (V_I - V_F) e^{-t/\tau} \)

\[
\frac{dV}{dt} = \frac{-1}{\tau} (V_I - V_F) e^{-t/\tau}
\]

\[
\frac{dV}{dt} \bigg|_{t=0} = \frac{-(V_I - V_F)}{\tau} \quad \text{volts/sec}
\]

0. The slope at \( t=0 \) can tell you the \( \tau \).
Things to remember about time constants

\[ e^{-2} = e^{-1} \cdot e^{-1} = 0.135 \]
\[ 0.368 \cdot 0.368 \]
\[ e^{-3} = e^{-1} \cdot e^{-1} \cdot e^{-1} = 0.0498 \]

So each additional time constant, the function drops down to 0.368 of the distance between where it started, and the asymptote for \( t \to \infty \)

What is a half-life? \( e^{-t} = 0.5 \), \( -t = \ln(0.5) \)
\[ t = 0.693 \pi \]

Energy in cap

\[ \frac{1}{2} CV^2, \frac{1}{2} C \left( V_0 e^{-t/\tau} \right)^2 = \frac{1}{2} CV_0^2 e^{-2t/\tau} \]

So, for \( t = \tau \), the energy is down to \( e^{-2} = 0.1353 \)
\[ t = \frac{\tau}{2}, \quad \frac{\tau}{4}, \quad \frac{\tau}{8} \quad \text{to} \quad e^{-4} = 0.018 \]

And so on

You'll always need to know the \( V_C(0^-) \) and \( i_L(0^-) \) because \( \frac{1}{2} CV^2 \) and \( \frac{1}{2} LI^2 \) can't change instantly unless the cap gets an \( \infty \) impulse of current, or the inductors gets an \( \infty \) impulse of voltage.

Get some initial \& final conditions for

Probs 7.5, 7.12, 7.9, 7.39, 7.45, 7.54