Problem 1. Your light dimmer circuit is shown. The source is DC rather than AC. The voltage across the 0.1\(\mu\)F capacitor is initially zero. The switch closes at \(t = 0\). When the capacitor voltage rises to 35V, the light bulb turns on and stays on. What resistance setting of the 250k\(\Omega\) potentiometer will cause the light bulb to turn on at \(t = 8\) milliseconds? Hint – ignore the light bulb resistance, and use capacitor voltage equation \(v_C(t) = V_F \left(1 - e^{-t/RC}\right)\).

\[ \frac{v_C}{V_F} = 1 - e^{-t/RC}, \quad C = 1 - \frac{V_C}{V_F}, \quad \frac{-t}{RC} = \ln\left(1 - \frac{V_C}{V_F}\right), \]

\[ R = \frac{-t}{C \ln\left(1 - \frac{V_C}{V_F}\right)} = \frac{-0.008}{(10^{-7}) \ln\left(1 - \frac{35}{100}\right)} = \frac{-0.008}{10^{-7}(0.4308)} = 185.7\,\text{K}\Omega \]

\[ R = 3.3\,\text{K}\Omega + R_{\text{pot}}, \quad R_{\text{pot}} = 185.7\,\text{K} - 3.3\,\text{K} = 182.4\,\text{K}\Omega \]

For \(t = 6\) msec, \( R = \frac{-0.006}{10^{-7}(0.4308)} = 139.3\,\text{K}, \quad R_{\text{pot}} = 136.0\,\text{K}\Omega \)

For \(t = 4\) msec, \( R = \frac{-0.004}{10^{-7}(0.4308)} = 92.9\,\text{K}, \quad R_{\text{pot}} = 89.6\,\text{K}\Omega \)
Problem 2. Your 18,000 \mu F capacitor is initially charged to 40V. Then, it is connected to a lossless DC-DC converter which takes the capacitor voltage and reduces it to a constant 20V. The load resistor is 500\,\Omega. How long does it take to use up one-half of the initial energy stored in the capacitor? Remember that the stored energy in a capacitor is $\frac{1}{2}CV^2$.

\[
W_{\text{cap}}(t=0) = \frac{1}{2}CV^2 = \frac{1}{2}(18000\times10^{-6})(40)^2 = 14.4 \, \text{J}
\]

\[W_{\text{cap}}(t=0) = \frac{1}{2}CV^2 = \frac{1}{2}(18000\times10^{-6})(40)^2 = 14.4 \, \text{J}
\]

Lossless DC-DC

So, $\frac{1}{2}W_{\text{cap}}(t=0) = P_{\text{load}} \Delta t$, \quad $\Delta t = \frac{\frac{1}{2}W_{\text{cap}}(t=0)}{P_{\text{load}}} = \frac{14.4}{0.8}$

\[\Delta t = 9.00 \, \text{sec} \]

For $R_L = 400\,\Omega$, $P_{\text{load}} = \frac{(20)^2}{400} = 1\,\text{W}$, $W_{\text{load}} = 1.0\,\text{J}$,

\[\Delta t = \frac{14.4}{1.0} = 7.20 \, \text{sec} \]

For $R_L = 300\,\Omega$, $P_{\text{load}} = \frac{(20)^2}{300} = 1.333\,\text{W}$,

\[\Delta t = \frac{14.4}{1.333} = 5.40 \, \text{sec} \]
Problem 3. The applied AC voltage is \(40\sin(\omega t)\) volts. Determine the average power to the 10\(\Omega\) resistor. Hint – recall that \(\sin^2 \theta = \frac{1 - \cos 2\theta}{2}\), and that \(V_{rms}^2 = \frac{1}{T} \int_{t_o}^{t_o+T} v^2(t)dt\). Also 20\(\Omega\), 30\(\Omega\).

If the diode was replaced by a wire, the average power to the resistor would be \(\frac{V_{rms}^2}{R} = \frac{(\frac{40\sqrt{2}}{2})^2}{R} = \frac{1600}{2(10)} = 80\text{W}\)

Exactly one-half of the average power is due to each half-cycle. So, if we take away every-other half-cycle with a diode, \(P_{avg}\) drops to \(\frac{1}{2}\). Thus \(P_{avg} = \frac{80}{2} = 40\text{W}\).

Also \(V_{rms}^2 = \frac{1}{T} \int_0^{T/2} V_p^2 \sin^2 \omega t dt = \frac{V_p^2}{2} \left[ \frac{T}{2} - \frac{1}{2\omega} \sin(2\omega t) \right]_0^{T/2} = \frac{V_p^2}{4\omega} \left[ T - \frac{1}{2} \sin(2\omega T) \right] = \frac{V_p^2}{4 \omega T} \sin(\pi) = \frac{V_p^2}{4\omega T} \sin(\pi) = \frac{V_p^2}{4}\)

\(V_{rms} = \frac{V_p}{\sqrt{2}}\), so \(P_{avg} = \frac{V_{rms}^2}{R} = \frac{(\frac{40\sqrt{2}}{2})^2}{10} = \frac{1600}{40} = 40\text{W}\)

Other versions, \(R_L = 20\Omega\), \(P_{avg} = 20\text{W}\)

\(R_L = 30\Omega\), \(P_{avg} = 13.33\text{W}\)
Problem 4. A perfect 60Hz voltage sinewave is fed into an amplifier, and the amplifier output contains 60Hz plus 3rd and 5th harmonics. The FFT is shown. The dB given are with respect to 1Vrms. Determine the THDv of the amplifier output.

\[
\text{THD}_V = \frac{\text{RMS of harmonics}}{\text{RMS of fund}} = \sqrt{\left(\frac{V_3}{V_1}\right)^2 + \left(\frac{V_5}{V_1}\right)^2}
\]

\[
\text{THD}_V^2 = \left(\frac{V_3}{V_1}\right)^2 + \left(\frac{V_5}{V_1}\right)^2
\]

20 \log_{10} \left(\frac{V_3}{V_1}\right) = -10 \text{dB}, \quad 20 \log_{10} \left(\frac{V_5}{V_1}\right) = -12 \text{dB}, \quad \frac{V_3}{V_1} = 10^{-1/2} = 0.316

\frac{V_5}{V_1} = 10^{-12/20} = 0.251

\text{THD}_V^2 = (0.316)^2 + (0.251)^2, \quad \text{THD}_V = 0.404

Other cases

\[
\begin{align*}
\frac{V_3}{V_1} & = 10^{-11/20} = 0.282, \quad \frac{V_5}{V_1} = 10^{-13/20} = 0.224 \\
\text{THD}_V & = 0.360
\end{align*}
\]

\[
\begin{align*}
\frac{V_3}{V_1} & = 10^{-12/20} = 0.251, \quad \frac{V_5}{V_1} = 10^{-14/20} = 0.200 \\
\text{THD}_V & = 0.321
\end{align*}
\]
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Problem 1. A MOSFET “fires” when the voltage across its gate capacitance $C_{GS}$ reaches 4V. Determine the value of $R$ in the firing circuit shown that will yield a 2 microsec delay between switch closing and MOSFET firing. Use $C_{GS} = 2$ nanoF.

![Circuit Diagram](image)

From the lab, you know that the 100kΩ is negligible because $R$ is small.

$$V_{GS} = 12 \left(1 - e^{-t/\tau}\right)$$

$$\frac{V_{GS}}{12} = 1 - e^{-t/\tau}, \quad e^{-t/\tau} = 1 - \frac{V_{GS}}{12}, \quad -\frac{t}{\tau} = \ln \left[1 - \frac{V_{GS}}{12}\right]$$

$$\tau = \frac{-\frac{t}{\ln \left[1 - \frac{V_{GS}}{12}\right]}}{\ln \left[1 - \frac{4}{12}\right]} = \frac{-2 \times 10^{-6}}{-0.405} = 4.93 \times 10^{-6}$$

$$\tau = RC, \quad R = \frac{\tau}{C} = \frac{4.93 \times 10^{-6}}{2 \times 10^{-9}} = 2470 \Omega$$

Other versions, 4 microsec delay, 4940 Ω

If you include the 100kΩ, you'll get a larger $R$ value than I did.

7410 | 100kΩ = 6.9 Ω (still pretty close to assumption)
Problem 2. A buck converter is shown. For the conditions given, carefully and neatly draw one cycle of the MOSFET current, showing current and time values at the corner points.

Input voltage = 120 V
Output voltage = 40 V
Frequency = 50 kHz
L = 100 microH
Power = 240 W

\[ D = \frac{V_{out}}{V_{in}} = \frac{40}{120} \]
\[ D = \frac{1}{3} \]

\[ T = \frac{1}{f} = \frac{1}{50 \times 10^3} \]
\[ T = 20 \mu s \]
\[ DT = 6.67 \mu s \]

When the switch is closed, the MOSFET current is \( i_L \).
Else, it is 0. The average inductor current is \( I_{out} \), and
\[ I_{out} = \frac{D}{V_{out}} = \frac{240}{40} = 6 \text{ A} \].

When the switch is closed, the inductor current is ramping up at
\[ \frac{V_{in} - V_{out}}{L} \]
\[ = \frac{120 - 40}{100} \frac{A}{\mu s} = 0.8 \frac{A}{\mu s} \text{ sec} \]. This lasts for DT sec, yielding a rise of
\[ (0.8)(\frac{1}{3})(20) = 5.33 \text{ A} \]. So the inductor current varies from \( 6 - \frac{5.33}{2} \) to \( 6 + \frac{5.33}{2} \)
\[ 3.33 \text{ A} \text{ to } 8.67 \text{ A} \].

Check:\n\[ \text{Avg}(i_{in}) = \left( \frac{8.67 + 3.33}{2} \right)(D) + (0)(1-D) = 2 \text{ A} \]

\[ (120 \text{V})(2 \text{A}) = 240 \text{W} \]
**Problem 2.** A buck converter is shown. For the conditions given, **carefully and neatly draw** the MOSFET current, showing current and time values at the corner points.

Input voltage = 120 V
Output voltage = 40 V
Frequency = 60 kHz
L = 100 microH
Power = 240 W

\[ T = \frac{1}{60 \text{kHz}} \]

= 16.67 µs

Rise lasts for \( DT = \frac{16.67}{3} = 5.56 \mu\text{sec} \)

Slope is 0.8 A/µsec → so rise is 4.44 µsec

Range of \( i_L \) is \( \left( \frac{6 - 4.44}{2} \right) \) to \( \left( \frac{6 + 4.44}{2} \right) \)

3.78 A \quad \text{to} \quad 8.22 A
Problem 2. A buck converter is shown. For the conditions given, carefully and neatly draw one cycle of the MOSFET current, showing current and time values at the corner points.

Input voltage = 120 V
Output voltage = 40 V
Frequency = 70 kHz
L = 100 microH
Power = 240 W

\[ T = \frac{1}{70 \text{kHz}} = 14.3 \mu\text{sec} \]

DT = 4.76 \mu\text{sec}

Rise time for DT = 4.76 \mu\text{sec}
Slope is 0.84 \mu\text{sec} \rightarrow so rise is 3.81 A

Range of \( i_L \) is \( \left( 6 - \frac{3.81}{2} \right) \) to \( \left( 6 + \frac{3.81}{2} \right) \)

4.10 A to 7.91 A
Problem 3. A buck/boost converter is shown. For the conditions given, carefully and neatly draw the current in capacitor C1. Show current and time values at the corner points.

\[ \frac{V_{out}}{V_{in}} = \frac{D}{1-D} = 3 \]

So, \[ 3 - 3D = D \]

Input voltage = 40 V
Output voltage = 120 V
Frequency = (50 kHz)
L1 = 100 microH
L2 = 500 microH
Power = 240 W

When the switch is closed, \( i_{C1} = -i_{L2} \). \( \text{Avg}(i_{L2}) = I_{out} = 2A \)

When the switch is open, \( i_{C1} = i_{L1} \). \( \text{Avg}(i_{L1}) = I_{in} = 6A \)

Plot \( i_{L1} \) & \( i_{L2} \), then get \( i_{C1} \).

Switch closed, \( \frac{di_{L1}}{dt} = \frac{40}{100} \mu A/sec = 0.4 A/\mu sec \), so \( \Delta i_{L1} = 6A \)

\[ \frac{di_{L2}}{dt} = \frac{40}{500} \mu A/sec = 0.08 A/\mu sec \], so \( \Delta i_{L2} = 1.2A \)

So \( i_{L1} \) is centered about 6A and varies \( \pm 3A \) (3A to 9A)

\( i_{L2} \) is centered about 2A and varies \( \pm 0.6A \) (1.4A to 2.6A)

Check \(-2(0.75) + 6(0.25) = 0\)

5/8
Problem 3. A buck/boost converter is shown. For the conditions given, carefully and neatly draw the current in capacitor C₁. Show current and time values at the corner points.

\[ T = 16.67 \mu \text{sec} \]
\[ DT = 12.5 \mu \text{sec} \]

\[ \Delta i_{L1} = (0.4 A/\mu \text{sec})(12.5 \mu \text{sec}) = 5 A, \text{ so } i_{L1} \text{ ranges from 3.5 A to 8.5 A} \]

\[ \Delta i_{L2} = (0.08 A/\mu \text{sec})(12.5 \mu \text{sec}) = 1 A, \text{ so } i_{L2} \text{ ranges from 1.5 A to 2.5 A} \]

Check \(-2(0.75) + 6(0.25) = 0\)

Full credit if DT and average \(i_{C1}\) during closed and open times are correct.
Problem 3. A buck/boost converter is shown. For the conditions given, **carefully and neatly draw one cycle** of the current in capacitor C₁. Show current and time values at the corner points.

\[ V_\text{in} \rightarrow \begin{array}{c} +v_{\text{L1}} \rightarrow \quad +v_{\text{C1}} \rightarrow \quad i_d \rightarrow \quad i_{\text{out}} \end{array} \]

\[ L_1 \quad C_1 \quad L_2 \quad C \quad V_\text{out} \]

\[ 0.01\Omega \]

**Input voltage = 40 V**

**Output voltage = 120 V**

**Frequency = 70 kHz**

**L1 = 100 microH**

**L2 = 500 microH**

**Power = 240 W**

\[ T = 14.3\,\mu\text{s} \]

\[ DT = 10.7\,\mu\text{s} \]

\[ \Delta i_{L1} = (0.4A/\mu\text{s}) \times (10.7\mu\text{s}) = 4.28A \]

so \( i_{L1} \) ranges from 3.86A to 8.14A

\[ \Delta i_{L2} = (0.08A/\mu\text{s}) \times (10.7\mu\text{s}) = 0.856A \]

so \( i_{L2} \) ranges from 1.57A to 2.43A

**Full credit if DT and average \( i_{C1} \) during closed and open times are correct**
Problem 4. Solar radiation measurements for a clear day are shown below. The vertical axis is $\text{W/m}^2$, and the horizontal axis is clock hour. For the day shown, a horizontal solar panel would have received 5.2 kWh/m$^2$ of incident solar energy. If the panel had been moved during the day by a two-axis mechanical tracker so that the panel always faced the sun directly, how many kWh/m$^2$ of incident solar energy would it have received for this day?

Use $\text{DN}$

$\text{A} = (0.9 \text{ kW/m}^2)(8 \text{ hrs}) = 7.2 \text{ kWh/m}^2$  
$8.9 = 72$ squares

$\text{B} = 5$ squares  
$\text{C} = 8\frac{1}{2}$ squares

$\text{D} = 5$ squares

$\text{E} = 4\frac{1}{2}$ squares

$\text{1/11} = 3$ squares

Total squares $= 98$

Each square is $0.1 \text{ kWh/m}^2$

So $= 9.8 \text{ kWh/m}^2$
Problem 1. Consider a MOSFET firing circuit that has a 12Vdc supply and a 1200Ω series gate firing resistor. The MOSFET turns “on” when $v_{GS}$ reaches 4V. For the situation described, this occurs at $t = 2\mu\text{sec}$.

If you use a DC supply voltage of 6V instead of 12V, what will be the new turn-on time? Note - work this problem using the capacitor charging expression $v_{GS} = V\left[1 - e^{-t/\tau}\right]$. Ignore the 100kΩ gate discharge resistor.

$$v_{GS} = V\left[1 - e^{-t/\tau}\right]$$

$$\frac{v_{GS}}{V} = 1 - e^{-t/\tau}, \quad e^{-t/\tau} = 1 - \frac{v_{GS}}{V},$$

$$\frac{-\frac{t}{\tau}}{\ln} = \ln\left[1 - \frac{v_{GS}}{V}\right], \quad t = -\tau \ln\left[1 - \frac{v_{GS}}{V}\right]$$

$\tau$ is the same regardless of $V$

So

$$\frac{t_{6V}}{t_{12V}} = \frac{-\tau \ln\left[1 - \frac{4}{6}\right]}{-\tau \ln\left[1 - \frac{4}{12}\right]} = \frac{\ln\left[1 - \frac{4}{6}\right]}{\ln\left[1 - \frac{4}{12}\right]}$$

$$\frac{t_{6V}}{t_{12V}} = \frac{1.0986}{0.4055} \quad \therefore t_{6V} = \frac{1.0986t_{12V}}{0.4055} = 5.42\mu\text{sec}$$

Other versions

$V=9$,

$$\frac{t_{9V}}{t_{12V}} = \frac{\ln\left[1 - \frac{4}{9}\right]}{\ln\left[1 - \frac{4}{12}\right]} = \frac{0.5878}{0.4055}$$

$$t_{9V} = 2.90\mu\text{sec}$$

$V=15$,

$$\frac{t_{15V}}{t_{12V}} = \frac{\ln\left[1 - \frac{4}{15}\right]}{\ln\left[1 - \frac{4}{12}\right]} = \frac{0.3102}{0.4055}$$

$$t_{15V} = 1.53\mu\text{sec}$$
Problem 2. The comparator in your PWM circuit acts as a grounding switch for the internal LEDs of two optocouplers in series. When "on," each LED has a forward drop of 1.5V. Assume that the design LED operating current is 25mA. Determine the resistance and average power rating of series biasing resistor $R$ in the 24V circuit. When determining the power rating, assume an on-off duty cycle of 50%.

$$R = \frac{21.0V}{0.025A} = 840 \Omega$$

If switch closed all the time, $P = I^2R = (0.025)^2(840) = 0.525 \text{ W}$

50% closed, 50% open (25 mA) (0 mA)

$$P_{av} = \frac{1}{2}(0.525) + \frac{1}{2}(0) = 0.263 \text{ W}$$

Other versions:

15 mA, $R = \frac{21V}{0.015} = 1400 \Omega$, $P_{av} = 0.1575 \text{ W}$

20 mA, $R = \frac{21V}{0.020} = 1050 \Omega$, $P_{av} = 0.210 \text{ W}$
Problem 3. Your inverter’s LC output filter consists of a 100µH series inductor and a 10µF shunt capacitor. Determine the capacitor’s rms current for the following condition:

- Inverter output is open circuited
- H-bridge output voltage to LC filter has 25Vrms of 1kHz, 10Vrms of 12kHz, and 20 Vrms of 24kHz.

\[
\begin{align*}
X_L(1\text{kHz}) &= 0.62852 \\
X_C(1\text{kHz}) &= -15.9252 \\
I_{1\text{kHz}} &= \frac{25\sqrt{2}}{0.628 - j15.92} = 1.635A \\
|I_{12\text{kHz}}| &= \frac{10}{7.54 - j1.33} = 1.610A \\
|I_{24\text{kHz}}| &= \frac{20}{15.08 - j0.663} = 1.387A \\
I_{\text{rms}} &= \sqrt{(1.635)^2 + (1.610)^2 + (1.387)^2} \\
I_{\text{rms}} &= 2.68A
\end{align*}
\]

Other Versions - \( I_{1\text{kHz}} \) same \((1.635A)\)

\[
\begin{align*}
|I_{15\text{kHz}}| &= \left| \frac{10}{9.42 - j1.06} \right| = 1.96A \\
|I_{30\text{kHz}}| &= \frac{20}{18.85 - j0.53} = 1.092A \\
I_{\text{rms}} &= \sqrt{(1.635)^2 + (1.96)^2 + (1.092)^2} \\
I_{\text{rms}} &= 2.30A
\end{align*}
\]

\[
\begin{align*}
|I_{18\text{kHz}}| &= \left| \frac{10}{11.31 - j0.884} \right| = 0.959A \\
|I_{36\text{kHz}}| &= \left| \frac{20}{22.62 - j0.442} \right| = 0.902A \\
I_{\text{rms}} &= \sqrt{(1.635)^2 + (0.959)^2 + (0.902)^2} \\
I_{\text{rms}} &= 2.10A
\end{align*}
\]
Problem 4. Consider the music testing station. If \( m_a = 1 \) for a 1kHz music note, what value of \( V_{dc} \) will provide 75W to the 32Ω speaker combination? (Note - assume that the impedance of your inverter, inverter output filter, and series DC blocking capacitors are negligible compared to the 32Ω speaker combination).

\[
V_{\text{Fund RMS}} = m_a \frac{V_{dc}}{\sqrt{2}}
\]

\[
P = \frac{V_{\text{Fund RMS}}}{R}, \quad V_{\text{Fund RMS}} = \sqrt{PR} = m_a \frac{V_{dc}}{\sqrt{2}}
\]

So,

\[
V_{dc} = \frac{\sqrt{2PR}}{m_a} = \frac{\sqrt{2(75)(32)}}{1} = 69.3 \text{ V},
\]

Other versions

100W \( \rightarrow \) \[ \frac{\sqrt{2(100)(32)}}{1} = 80V \]

120W \( \rightarrow \) \[ \frac{\sqrt{2(120)(32)}}{1} = 87.6 \text{ W} \]
Problem 5. With a sinusoidal input, the output of your music amplifier has 3rd and 5th harmonics that are 24dB and 29dB down from the fundamental. Compute the THDv.

\[ 20 \log_{10} \left( \frac{V_3}{V_1} \right) = -24 \text{dB} \]

\[ 10 \log_{10} \left( \frac{V_3}{V_1} \right) = \frac{-24 \text{dB}}{20} \]

\[ \frac{V_3}{V_1} = 10 \left[ \frac{-24 \text{dB}}{20} \right] = 0.0631 \]

\[ \frac{V_5}{V_1} = 10 \left[ \frac{-29 \text{dB}}{20} \right] = 0.0355 \]

\[ \text{THD} = \sqrt{(0.0631)^2 + (0.0355)^2} \Rightarrow 7.24\% \]

Other Versions

\[ \frac{V_3}{V_1} = 10 \left[ \frac{-28.7 \text{dB}}{20} \right] = 0.0398 \]

\[ \Rightarrow 4.57\% \]

\[ \frac{V_3}{V_1} = 10 \left[ \frac{-33 \text{dB}}{20} \right] = 0.0224 \]

\[ \Rightarrow 4.57\% \]

\[ \frac{V_3}{V_1} = 10 \left[ \frac{-30 \text{dB}}{20} \right] = 0.0316 \]

\[ \Rightarrow 3.63\% \]

\[ \frac{V_5}{V_1} = 10 \left[ \frac{-35 \text{dB}}{20} \right] = 0.0178 \]
Problem 1. The periodic waveform shown is applied to a 100Ω resistor. What value of $\alpha$ yields 50W average power to the resistor?

Segmented waveform - weight the $\text{rms}^2$ values of each portion by their fraction of the cycle.

\[
V_{\text{rms}}^2 = \alpha \left[ V_{\text{avg}}^2 + \frac{1}{12} V_{\text{pp}}^2 \right] + (1-\alpha)(0)^2
\]

\[
P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} = \alpha \frac{V_{\text{avg}}^2 + \frac{1}{12} V_{\text{pp}}^2}{R}
\]

\[
\alpha = \frac{P_{\text{avg}} R}{V_{\text{avg}}^2 + \frac{1}{12} V_{\text{pp}}^2} = \frac{(50)(100)}{\left( \frac{50^2 + \frac{1}{12}(150)^2}{75^2 + \frac{1}{12}(150)^2} \right)} = \frac{5000}{5625 + 1875} = \frac{5000}{7500}
\]

\[
\alpha = \frac{2}{3}
\]
Problem 2. The output of an amplifier contains a desired 10Vrms fundamental component, but it also has undesired 5th and 7th harmonic components that are both 25dB down from the fundamental. What are the rms magnitudes of the 5th and 7th harmonics?

\[ 20 \log_{10} \left( \frac{V_5}{V_1} \right) = -25 \text{dB} \]

\[ \log_{10} \left( \frac{V_5}{V_1} \right) = \frac{-25}{20} \]

\[ \frac{V_5}{V_1} = 10^{\frac{-25}{20}} = 0.0562 \]

So \[ V_5 = (10)(0.0562) = 0.562 \text{V}_{\text{rms}} \]

\[ V_7 = V_5 \]

(I should have asked for THDv)
Problem 3. Consider a transformerless DBR that operates directly on 120Vrms, 60Hz. The DBR powers a PC that draws a constant 100W. The PC employs a regulated internal DC-DC converter to change the DBR cap voltage to reach the required voltages needed by the PC.

Estimate the μF/100W ratio that will give you 10 cycles (of 60Hz) ride-through capability when a power outage occurs. Make a reasonable assumption concerning the normal DBR capacitor voltage, and then assume that the PC's DC-DC converter trips off when the DBR capacitor voltage drops to 60% of nominal.

\[ V_p \approx (120\sqrt{2})(0.95) = 161.2\, V \]

Energy given up by cap

\[ \frac{1}{2} C \left( V_p^2 - (0.60 V_p)^2 \right) = (100\, \text{W})(\frac{10}{60}\, \text{s}) \]

\[ C = \frac{200\left(\frac{1}{6}\right)}{(161.2)^2(1-0.36)} = 2004\, \mu\text{F} \quad \text{for 100 W} \]

So the ratio is proportional to P

20.04 μF/1W, or \( \boxed{2004\, \mu\text{F}/100\, \text{W}} \)

\[ 1.10 \rightarrow 1809\, \mu\text{F} \]
\[ 0.90 \rightarrow 2233\, \mu\text{F} \]

Using e^{-τC} to reduce the estimate to about 1300 μF/100W

(don't use e^{-τC})

Using \( \frac{200(\frac{1}{6})}{(161.2)^2(1-0.6)} \) → 3207 μF
Problem 4. A simple DC-DC converter is shown. The output is a constant power load.

4a. Assuming continuous conduction in L, and ripple free $V_{out}$ and $I_{out}$, draw the "switch closed" and switch open" circuits and use them to develop the $\frac{V_{out}}{V_{in}}$ equation.

4b. Consider the case where the converter is operating at 50kHz, $V_{in} = 40V$, $V_{out} = 120V$, $P = 240W$. Components $L = 100\mu H$, $C = 1500\mu F$. Carefully sketch the inductor and capacitor currents on the graph provided.

4c. Use the graphs to determine the inductor’s rms current, and the capacitor’s peak-to-peak current.

4d. Use the graphs to determine the capacitor’s peak-to-peak ripple voltage.

4a. Avg voltage across $L = 0$

$$\frac{1}{T}(V_{in}D - V_{out}(1-D)) = 0$$

$V_{in}D = V_{out}(1-D)$,

$V_{out} = V_{in} \frac{D}{1-D}$

Buck/Boost

4b. $I_{in} = \frac{240W}{40V} = 6A$, $I_{out} = 2A$,

Switch closed, $\frac{dI_L}{dt} = \frac{V_{in}}{L} = \frac{40}{100} \text{ A}/\mu \text{sec} = 0.4 \text{ A}/\mu \text{sec}$

$\frac{V_{out}}{V_{in}} = \frac{D}{1-D}$, $\frac{120}{40} = \frac{D}{1-D}$, $3(1-D) = D$, $3 - 3D = D$, $3 = 4D$, $D = \frac{3}{4}$

$f = 50kHz \rightarrow T = 20 \mu \text{sec}$

$DT = 15 \mu \text{sec}$

$0.4 \text{ A}/\mu \text{sec} \cdot 15 \mu \text{sec} = 6 \text{ A} \text{ Rise}$

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With the switch closed, the C provides $\mathbf{- I_{load}} = \frac{-240}{120} = -2A$

With the switch open, $\mathbf{I_C} = \mathbf{I_L} - \mathbf{I_{load}}$

Check C & P replenishment

Test $(-2A)(DT) + (6A)(1-DT) = 0$ ?

$(-2A)\left(\frac{3}{4}\right) + (6A)\left(\frac{1}{4}\right) = -\frac{6}{4} + \frac{6}{4} = 0 \quad \text{(OK)}$

4c. $\mathbf{I_{LRMS}^2 = 8^2 + (\frac{1}{2})6^2 = 64 + 3 = 67, \quad I_{LRMS} = 8.19A}$

$\mathbf{I_{peak-to-peak}} = 9A - (-2A) = \boxed{11A}$

4d. $\mathbf{\Delta Q = C \Delta V, \quad \Delta V = \frac{\Delta Q}{C} = \frac{(2A)(15\mu s)}{1500\mu F} = 0.02V}$
Problem 5. As you all saw, Sunday was a brilliant solar day here in Austin. Consider a PV installation that has 60° tilt, and 225° azimuth (i.e., facing southwest). Use the following equation,

\[ P_{\text{incident}} = \left[ DH + \frac{(GH - DH)}{\cos(\theta_{\text{zenith}})} \cdot \cos(\beta_{\text{incident}}) \right] W / m^2, \]

and the graphs on the following page to estimate

5a. the maximum incident solar power density on the panels (in W/m²), and **About 1000 W/m²**

5b. the time at which the maximum occurs. **3 p.m.**

\[
\begin{align*}
15:00 & \quad P_{inc} = \left[ 30 + \frac{(445-30)}{\cos(65^\circ)} \cdot \cos(7.5^\circ) \right] = 1004 \text{ W/m}^2 \\
14:30 & \quad \left[ 30 + \frac{(510-30)}{\cos(61)} \cdot \cos(11) \right] = 1002 \text{ W/m}^2 \\
14:00 & \quad \left[ 30 + \frac{(555-30)}{\cos(58)} \cdot \cos(17) \right] = 977 \text{ W/m}^2 \\
15:30 & \quad \left[ 30 + \frac{(370-30)}{\cos(59)} \cdot \cos(90) \right] = 967 \text{ W/m}^2 \\
13:30 & \quad \left[ 35 + \frac{(590-35)}{\cos(56)} \cdot \cos(23) \right] = 949
\end{align*}
\]
Sun Zenith Angle (Top Curve), and Incident Angle on Panel (Bottom Curve), for Dec. 16
(Panl Tilt and Azimuth = 60 and 225 Degrees, Respectively)

Global Horizontal (Top Curve), and Diffuse Horizontal (Bottom Curve), for Dec. 16