MIMO Cognitive Radio With Channel Covariance Feedback

Liang Dong
Department of Electrical and Computer Engineering
Baylor University, Waco, Texas 76798-7356, USA

Abstract—In MIMO cognitive radio networks, the secondary users are in cognizant of the spatial channels toward the primary users. Based on the knowledge of the interference channel toward the primary receivers and the channel toward its intended receiver, the secondary transmitter adjusts its transmission spatial spectrum in order to maximize its own information rate while limiting interference to the primary receivers. This paper considers the practical case where only the covariance of the time-varying channel matrix can be fed back to the secondary transmitter through a low-rate feedback channel. The optimal transmission is derived when either transmit power constraint or interference power constraint is imposed. Under both constraints, a suboptimal transmission scheme is proposed for the cognitive radio network. Simulation results show the average information rate per secondary user at the Nash equilibrium, which is compared with the rate in the case of perfect feedback of the channel state information.

I. INTRODUCTION

Cognitive radio is an efficient wireless communication technique that enhances access to the radio spectrum. In a cognitive radio network, the secondary user of the spectrum is allowed to access the licensed band of the primary user only in adherence to strict access policies so as not to degrade the primary user’s quality of service. The secondary transmitter traditionally senses spectrum holes in the primary communication and transmits accordingly to minimize interference at the primary receiver [1]. When multiple antennas are used at both the transmitters and the receivers, multiple-input multiple-output (MIMO) channel links are established. The secondary transmitter can cognize spatial holes toward the primary receiver and transmit accordingly to achieve maximum throughput while maintaining the interference to the primary receiver within a limit [2].

As multiple non-cooperative secondary users share a single-frequency channel advertised by the primary user, the game theory can be applied to the cognitive radio network [3]. The multi-user interference and noise are treated as colored noise at the secondary receiver, and the whitened channel matrix is fed back to the secondary transmitter. Each secondary user competes with others and tries to maximize individual transmission rate in a non-cooperative manner. In addition, the secondary transmitter usually can not receive and decode the primary user’s message. Therefore, no interference cancellation can be performed.

Most work to date regards the cognition of the MIMO spatial channel as perfect feedback of the channel state information (CSI). For time-varying wireless channels, the frequent feedback of the CSI is costly. In practice, only statistical information of the primary receiver’s interference channel and the secondary user’s channel can be fed back to the secondary transmitter. With channel covariance feedback, multiple-eigenmode transmission and reception can maximize the ergodic capacity [4], [5]. In this paper, we consider the practical scenario where only the covariance of the whitened secondary user channel matrix and the covariance of the primary receiver interference channel matrix can be fed back.

The optimal transmission in terms of transmit spatial spectrum (across multiple transmit antennas) is derived when either transmit power constraint or interference power constraint is imposed. It is discussed as under what condition both constraints need to be considered. A suboptimal transmission scheme is proposed for cognitive radio network under both constraints. Simulation results reveal the average information rate per secondary user at the Nash equilibrium, which is promising performance compared with the case of perfect CSI feedback. The proposed transmission schemes are practical and efficient with little tradeoff in achievable throughput.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cognitive radio network with $P$ primary receivers and $K$ pairs of secondary transmitter and receiver. The primary and secondary users share the same radio channel resources such as time and frequency. Multiple antennas are equipped at the primary receivers and the secondary transmitters and receivers. Each primary receiver has $M_{R_p}$ receive antennas and each secondary user has $M_{T_s}$ transmit and $M_{R_s}$ receive antennas. This gives MIMO peer-to-peer links of secondary users and between the secondary transmitters and the primary receivers. At the $i$th secondary receiver, the baseband signal can be modeled as

$$y_i = H_{ii}x_i + \sum_{j=1, j \neq i}^{K} H_{ij}x_j + z_i \quad (1)$$

where $y_i \in \mathbb{C}^{M_{R_p}}$ is the received signal vector, $x_i \in \mathbb{C}^{M_{T_s}}$ and $x_j \in \mathbb{C}^{M_{T_s}}$ are the signal vectors transmitted at secondary transmitter $i$ and secondary transmitter $j$, respectively. $H_{ii} \in \mathbb{C}^{M_{R_s} \times M_{T_s}}$ is the channel matrix between secondary transmitter $i$ and its intended receiver, $H_{ij} \in \mathbb{C}^{M_{R_s} \times M_{T_s}}$ is the cross-channel matrix between secondary transmitter $j$ and secondary receiver $i$, and $z_i \in \mathbb{C}^{M_{R_s}}$ is a zero-mean circularly symmetric complex Gaussian noise vector.
that contains both thermal noise and possible interference generated by the primary users. Let $Q_i$ denote the transmit covariance matrix of secondary transmitter $i$, $Q_i = E[x_i x_i^H]$. It is assumed that the Gaussian codebook with infinitely many codewords is used for the transmit symbols and the expectation is taken over the entire codebook. Therefore, $x_i$ is circularly symmetric complex Gaussian such that $x_i \sim CN(0, Q_i)$. The covariance matrix is positive semi-definite such that $Q_i \succeq 0$.

The transmit power at the secondary transmitter $i$ is limited by its own transmit power constraint $P_i$, i.e. $\text{Tr}(Q_i) \leq P_i$. At the $p$th primary receiver, the interference power over all receive antennas that generated by secondary transmitter $i$ is $\text{Tr}(G_{pi} Q_i G_{pi}^H)$, where $G_{pi} \in \mathbb{C}^{M_{rp} \times M_{tr}}$ is the cross-channel matrix between secondary transmitter $i$ and primary receiver $p$.

From an information-theoretical perspective, the capacity loss of the primary user due to the secondary user transmission is well regulated by the interference power constraint over all receive antennas [2]. Therefore, we consider the interference power over all receive antennas instead of interference power on each individual receive antenna. The interference power constraint is $\text{Tr}(G_{pi} Q_i G_{pi}^H) \leq \Gamma_{p,i}$, where $\Gamma_{p,i}$ is determined in advance for each secondary transmitter $i$ such that $\sum_{i=1}^{K} \Gamma_{p,i}$ is below the interference temperature limit at primary receiver $p$ [1], [3].

It is assumed that secondary receiver $i$ has perfect CSI of $H_{ii}$ and the covariance matrix of the multi-user interference-plus-noise $Q_i = \sum_{j \neq i} H_{ij} Q_i H_{ij}^H + R_{zi}$, where $R_{zi}$ is the noise covariance matrix. For fast time-varying channel and feedback channel with limited bandwidth, the partial CSI in terms of channel covariance of $H_{ii}$ and the interference-plus-noise covariance $R_i$ averaged over varying $H_{ij}$’s are fed back to the secondary transmitter. Suppose that the channel statistics of $G_{pi}$ from secondary transmitter $i$ to primary receiver $p$ can also be acquired by the secondary transmitter. In practice, the secondary transmitter can achieve this by continuously sensing the transmitted signal from the primary receiver if the primary user employs time-division duplexing (TDD) [2]. Usually, the secondary users do not know the message of the primary users, and there is no coordination among the secondary users. Therefore, no interference cancellation can be performed in secondary transmissions. Based on the feedback of the channel covariance and the interference-plus-noise covariance, secondary transmitter $i$ adapts its spatial spectrum $Q_i$, in order to maximize the information rate under its own transmit power constraint and the interference power constraints at the primary receivers. The optimal transmission problem for secondary user $i$ can be formulated as

$$\max_{Q_i \succeq 0} R(Q_i)$$
subject to
$$\text{Tr}(Q_i) \leq P_i$$
$$\text{Tr}(G_{pi} Q_i G_{pi}^H) \leq \Gamma_{p,i}, \quad p = 1, \ldots, P$$

where $R(Q_i) = E[\log_2 |I + R_i^{-1} H_{ii} Q_i H_{ii}^H|]$ is the ergodic maximum information rate with the transmit covariance matrix $Q_i$ averaged over $H_{ii}$.

### III. TRANSMITTER OPTIMIZATION WITH CHANNEL AND INTERFERENCE-PLUS-NOISE COVARIANCE FEEDBACK

Without the constraints due to the interference power limits, the system is simplified to a non-cooperative multiuser MIMO system. A strategic game can be formulated in which the players are the secondary transmitters and the payoffs are the information rates of the secondary user links. The game is given by

$$(G_1): \max_{Q_i \succeq 0} R(Q_i)$$
subject to
$$\text{Tr}(Q_i) \leq P_i, \quad i = 1, 2, \ldots, K. \quad (3)$$

The link information rate is equivalently given by

$$R(Q_i) = E[\log_2 |I + R_i^{-1} H_{ii} Q_i H_{ii}^H R_i^{-1/2}|]$$

where the expectation is taken over all $H_{ii}$’s and $H_{ij}$’s.

Suppose that secondary receiver $i$ can acquire the covariance of the whitened channel matrix $R_i^{-1/2} H_{ii}$ when the other secondary transmitters choose their spatial spectra $Q_{-i} = \{Q_j\}_{j=1, j \neq i}$. This covariance is relatively stable and can be fed back to secondary transmitter $i$ using a low-bandwidth channel. The channel covariance contains the spatial directions through which the transmission can achieve the link capacity. Decompose the channel covariance matrix as $E[H_{ii}^H R_i^{-1} H_{ii}] = U_{\Sigma_i} \Lambda_{\Sigma_i} U_{\Sigma_i}^H$, where $U_{\Sigma_i}$ is a unitary matrix with the eigenvectors and $\Lambda_{\Sigma_i}$, a diagonal matrix with the positive eigenvalues in descending order. Given spectral profile $Q_{-i}$, the capacity-achieving spatial spectrum for secondary transmitter $i$ is the waterfilling solution as

$$Q_i^* = U_{\Sigma_i} (\mu_i I - \Lambda_{\Sigma_i}^{-1})^+ U_{\Sigma_i}^H$$

where $(\bullet)^+$ denotes $\max(0, \bullet)$ and $\mu_i$ is chosen to satisfy

$$\text{Tr}((\mu_i I - \Lambda_{\Sigma_i}^{-1})^+) = P_i.$$ The transmit directions are determined by $U_{\Sigma_i}$ that are indeed the eigenvectors of the transmitter side channel correlation matrix [4], [5].

In strategic game $G_1$ with $K$ players (transmitters), each one competes against others by choosing his strategy (optimal transmission spatial spectrum $Q_i$, $i = 1, \ldots, K$). A Nash equilibrium is reached when each player (transmitter), given the strategies of other players ($Q_{-i}$), does not increase his payoff (link information rate) by unilaterally changing his own strategy ($Q_i$). Clearly, the spectral profile $\{Q_i^*\}_{i=1}^K$ that satisfies a set of waterfilling solutions (5) is a Nash equilibrium. The existence of the equilibrium and the sufficient conditions (on interference) guaranteeing its uniqueness were derived by Arslan et al. [6] and Scutari et al. [7].

### IV. GAME WITH INTERFERENCE-POWER CONSTRAINTS

With interference power constraints imposed by the primary receivers, the strategic game for optimal secondary user transmissions can be formulated as

$$(G_2): \max_{Q_i \succeq 0} R(Q_i)$$
subject to
$$\text{Tr}(Q_i) \leq P_i$$
$$\text{Tr}(G_{pi} Q_i G_{pi}^H) \leq \Gamma_{p,i}, \quad p = 1, \ldots, P \quad (6)$$
for $i = 1,2,\ldots,K$. Let us consider the case where there is at most one primary receiver imposing interference power constraint on each secondary transmitter. For secondary transmitter $i$, the constraint becomes $\text{Tr}(G_i Q_i G_i^H) \leq \Gamma_i$. Suppose that the interference channel $G_i$ is full column rank, i.e. $M_{R_p} \geq M_{T_x}$ and $\text{rank}(G_i) = M_{T_x}$, and the covariance matrix of the interference channel can be acquired by the primary receiver and fed back to secondary transmitter $i$. Decompose the interference channel covariance matrix as $E[G_i^H G_i] = U_{G_i} A_{G_i} U_{G_i}^H$, where $U_{G_i}$ is a unitary matrix with the eigenvectors and $A_{G_i}$ a diagonal matrix with the positive eigenvalues in descending order. For an invariant channel $G_i$, we have

$$\text{Tr}(G_i Q_i G_i^H) = \text{Tr}(A_{G_i} U_{G_i}^H Q_i U_{G_i}) = \text{diag}(A_{G_i})^T \text{diag}(U_{G_i}^H Q_i U_{G_i}).$$ (7)

Let $\lambda_{\min}(A_{G_i})$ denotes the last diagonal element in $A_{G_i}$ and $\lambda_{\max}(A_{G_i})$ the first diagonal element in $A_{G_i}$. It follows that $\lambda_{\min}(A_{G_i}) \text{Tr}(Q_i) \leq \text{Tr}(G_i Q_i G_i^H) \leq \lambda_{\max}(A_{G_i}) \text{Tr}(Q_i).$ (8)

Therefore, if the interference power constraint $\Gamma_i \geq \lambda_{\max}(A_{G_i}) P_i$, the transmit power constraint $\text{Tr}(Q_i) \leq P_i$ dominates and the problem is reduced to game $G_i$. If the interference power constraint $\Gamma_i \leq \lambda_{\min}(A_{G_i}) P_i$, the interference power constraint $\text{Tr}(G_i Q_i G_i^H) \leq \Gamma_i$ dominates.

For the later case, it can be shown that a Nash equilibrium of game $G_i$ exists and it is the solution to the set of nonlinear matrix-valued equations ($i = 1,2,\ldots,K$):

$$Q_i = WF_i(U_{G_i}^H U_{G_i}^T A_{G_i} U_{G_i}^H U_{G_i}^T A_{G_i}^{-1} U_{G_i}^H G_i).$$ (14)

where $WF_i(X)$ is the MIMO waterfilling operator as defined in (10) and the water level $\mu_i$ is chosen to satisfy $\text{Tr}((\mu_i I - A_{G_i}^{-1})) = \Gamma_i$. Therefore, the optimal spatial spectrum for each secondary transmitter can be written as ($i = 1,2,\ldots,K$)

$$Q_i = U_{G_i} A_{G_i}^{-1} Q_i U_{G_i}^H = U_{G_i} A_{G_i}^{-1} WF_i(U_{G_i}^H U_{G_i}^T A_{G_i} U_{G_i}^H U_{G_i}^T A_{G_i}^{-1} U_{G_i}^H G_i).$$ (15)

Regarding the waterfilling operator as a projection onto a convex set $Q_i = \{X \in \mathbb{C}^{M_{T_x} \times M_{T_x}} : X \succeq 0, \text{Tr}(X) = \Gamma_i\}$, we can obtain sufficient conditions guaranteeing the uniqueness of the Nash equilibrium (similar to the work done by Scutari et al. [3]).

V. SUBOPTIMAL SOLUTION OF GAME WITH BOTH TRANSMIT AND INTERFERENCE POWER CONSTRAINTS

Both the transmit power constraint and the interference power constraint need to be considered, if the interference power constraint is within the range

$$\lambda_{\min}(A_{G_i}) P_i \leq \Gamma_i < \lambda_{\max}(A_{G_i}) P_i.$$ (16)

The strategic game for optimal secondary user transmission can be formulated as

$$\max_{Q_i \succeq 0} R(Q_i),$$ (17)

subject to $\text{Tr}(Q_i) \leq P_i$,

$$\text{Tr}(A_{G_i} U_{G_i}^H Q_i U_{G_i}) \leq \Gamma_i,$$ for $i = 1,2,\ldots,K$. We adopt a suboptimal solution to game $G_i$, $A_{G_i}$ is a diagonal matrix with the positive eigenvalues in descending order, that is $A_{G_i} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{M_{T_x}})$ and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{M_{T_x}}$. Suppose that $\lambda_{m+1}(A_{G_i}) \leq \Gamma_i \leq \lambda_{m}(A_{G_i})$, $m = 1,\ldots,M_{T_x} - 1.$ (18)

Let $U_{G_i} \in \mathbb{C}^{M_{T_x} \times (M_{T_x} - m)}$ be the semi-unitary matrix with the right $(M_{T_x} - m)$ columns of $U_{G_i}$, which correspond to the $(M_{T_x} - m)$ smallest eigenvalues in $A_{G_i}$. Let $P_{U_{G_i}}$ be the orthogonal projection onto the subspace spanned by the columns of $U_{G_i}$, i.e. $P_{U_{G_i}} = U_{G_i} U_{G_i}^H$. In order to avoid transmission in the directions of interference channel corresponding to the $m$ largest eigenvalues [2], the channel $H_i$ is modified as $H_i P_{U_{G_i}}$. The maximum information rate becomes

$$R(Q_i) = E[\log_2 (1 + \text{Tr}(A_{G_i} P_{U_{G_i}} H_i^H R_i^{-1} H_i P_{U_{G_i}} Q_i)]$$ (19)

where $R_i = \sum_{j \neq i} H_j P_{U_{G_i}} Q_i P_{U_{G_i}} H_j H_j^H + R_{ij}$. A Nash equilibrium exists for the strategic game with modified channels and it is given by ($i = 1,2,\ldots,K$)

$$Q_i^* = \text{WF}_i(U_{G_i}^H U_{G_i}^T A_{G_i} U_{G_i}^H U_{G_i}^T A_{G_i}^{-1} U_{G_i}^H G_i U_{G_i}) U_{G_i},$$ (20)

Here, $WF_i(\bullet)$ is the reduced-dimension waterfilling operator. The water level $\mu_i$ is chosen to satisfy $\text{Tr}((\mu_i I - A_{G_i}^{-1})) = \Gamma_i.$
The sufficient conditions that guarantee the uniqueness of the Nash equilibrium can be derived as well. It can be shown that

$$\text{Tr}(A_{G_i} U_{G_i}^H, Q_i, U_{G_i}) \leq \lambda_{m+1}(A_{G_i}) \text{Tr}(Q_i) \leq \Gamma_i.$$  

(21)

When there are $P$ primary receivers imposing interference power constraints on the secondary transmitters, secondary transmitter $i$ has the constraints

$$\text{Tr}(A_{G_p} U_{G_p}^H, Q_i, U_{G_p}) \leq \Gamma_{pi}, \quad p = 1, \ldots, P.$$  

(22)

We collect the constraints from the primary receivers $\theta \in \Theta \subset [1, \ldots, P]$ such that $\Gamma_{\theta_i} < \lambda_1(A_{G_{\theta i}})P_i, \forall \theta$. Decompose the weighted sum of the covariance matrices of these interference channels as

$$\frac{1}{|\Theta|} \sum_{\theta \in \Theta} \frac{1}{\Gamma^{\theta_i}} U_{G_{\theta i}} A_{G_{\theta i}} U_{G_{\theta i}}^H = V_{G_i} D_{G_i} V_{G_i}^H.$$  

(23)

where $|\Theta|$ is the cardinality of set $\Theta$. $V_{G_i}$ is a unitary matrix with the eigenvectors and $D_{G_i}$ a diagonal matrix with the positive eigenvalues in descending order. A necessary condition of the interference power constraints is

$$\text{Tr}(D_{G_i} V_{G_i}^H, Q_i, V_{G_i}) \leq 1.$$  

(24)

A suboptimal solution can be derived similar to the previous treatment of the single primary receiver case. With $\lambda_{t+1}(D_{G_i}) \leq P_i^{-1} < \lambda_1(D_{G_i})$, we choose the right $(M_{F_i} - t)$ columns of $V_{G_i}$ to form a semi-unitary matrix $V_{G_i} \in C^{M_{F_i} \times (M_{F_i} - t)}$ and modify the secondary channel as $H_{G_i} V_{G_i}, V_{G_i}^H$. Due to independence of the interference channels, this method tends to reduce transmission in the dominant direction of each interference channel. It is unlikely but there can be interference beyond the thresholds $\Gamma_{pi}$'s at some primary receivers. We can further adjust the power allocation of secondary transmitter $i$ to satisfy every individual interference power constraint.

VI. NUMERICAL RESULTS

A cognitive radio scenario is simulated where there are $K = 3$ pairs of secondary transmitter and receiver and $P = 1$ primary receiver. Each secondary user pair has four transmit and four receive antennas and the primary receiver has four receive antennas. This provides a $4 \times 4$ MIMO channel between the secondary transmitter and receiver and a $4 \times 4$ MIMO interference channel between the secondary transmitter and the primary receiver.

The channels of secondary users $\{H_{ui}\}$ and the interference channels to the primary receiver $\{G_i\}$ are normalized, and the cross-channels $\{H_{ij}\}$ are normalized and adjusted (basically attenuated) according to the signal-to-interference ratio (SIR). With a zero-mean unit-variance noise, the transmit power is regulated according to the signal-to-noise ratio (SNR). For instance, $P$ chosen from 1 to 100 corresponds to a SNR from 0 dB to 20 dB. All the channels are time-varying. Considering fade correlations at the transmitter and the receiver, we model a MIMO channel as $H = \Sigma^{1/2} H_w \Sigma_i^{1/2}$, where $H_w$ is a matrix of i.i.d. zero-mean unit-variance complex Gaussian elements, $\Sigma_r$ and $\Sigma_t$ represent the fade correlations at the receive and transmit antennas, respectively.

Suppose that the transmit power constraints $\{P_i\}$ are the same for $K = 3$ secondary transmitters. With only the transmit power constraints, Fig. 1 shows the average information rate per user at the Nash equilibrium with various SNR, i.e. various $P$, and various SIR with interference from other secondary transmitters. The information rate increases with higher SNR and SIR. The dashed-line curves correspond to optimal transmission with perfect CSI feedback while the solid-line curves correspond to optimal transmission with covariance feedback of the channel and the interference-plus-noise. The covariance feedback is less demanding, but it trades off with some information rate.
Suppose that the interference power constraints \( \{ \Gamma_i \} \) are the same for \( K = 3 \) secondary transmitters. With only the interference power constraints, Fig. 2 shows the average information rate per user at the Nash equilibrium with various SIR. The primary user interference constraint is set as \( \Gamma = 1, 0.1, 0.01 \). The information rate increases with higher SIR and larger \( \Gamma \). The dashed-line curves correspond to optimal transmission with perfect feedback of channels \( \mathbf{H} \) and \( \mathbf{G} \) while the solid-line curves correspond to optimal transmission with covariance feedback of both channels. The seemingly large information rate is due to the large transmit power of each secondary user. Without any constraint, the average SNR in this simulation reaches 69.9 dB, 55.7 dB, and 41.4 dB for \( \Gamma = 1, 0.1, 0.01 \) respectively with perfect CSI feedback and 23.9 dB, 17.4 dB, and 5.6 dB respectively with channel covariance feedback. The large difference in simulated transmit powers of these two cases contributes to the additional gap between the information rates (dashed-line versus solid-line). For the covariance feedback case, Fig. 3 shows the values of \( \text{Tr}(\mathbf{G}_i \mathbf{Q}_i \mathbf{G}_i^H) \) over varying channels when the average primary receiver interference constraint is set as \( \Gamma = 1 \). There are quite a few small individual values with limited transmissions that cause the decrease in the information rate.

With both transmit power and interference power constraints, Fig. 4 shows the average information rate per user at the Nash equilibrium using the suboptimal transmission scheme. The SNR is elaborated as 5 dB in order to satisfy (16). The optimal transmission at SNR = 5 dB with no interference constraint is also plotted for comparison (denoted as \( \Gamma = \infty \)). With relatively large interference limit, the performance of the suboptimal transmission is close to the optimal one. With small interference limit, the information rate is low because the valid subspace may become null which results in no transmission at all.

VII. Conclusion

In a MIMO cognitive radio network, transmission schemes of the secondary users in terms of adapting spatial spectra are proposed with practical feedback of the channel covariance. When either the transmit power constraint or the interference channel constraint is imposed, the optimal transmission is derived using a waterfilling solution. With both constraints, a suboptimal transmission is proposed that uses a waterfilling solution on a modified channel. The modified channel is the orthogonal projection of the whitened secondary user channel onto the subspace that nulls the dominant spatial directions toward the primary receivers. The performance in the average information rate per user at the Nash equilibrium is promising compared with the case of perfect CSI feedback.

REFERENCES