

# OPPORTUNISTIC TRANSMISSION SCHEDULING FOR MULTIUSER MIMO SYSTEMS

Liang Dong, Teng Li, and Yih-Fang Huang

Department of Electrical Engineering, University of Notre Dame  
Notre Dame, IN 46556, USA

## ABSTRACT

An opportunistic transmission scheduling scheme is proposed to make better use of the multiuser diversity gain in a MIMO system. The performance of a MIMO link is quantified by its information-theoretic capacity. Even though the base station has no knowledge of the transmission channel, the proposed algorithm generates a transmission randomizing matrix according to the distribution of the channel, so that it is likely that some user is near the “water-filling” configuration. The mobile users measure and feedback the channel quality. Based on this information, the base station schedules transmission to the user whose instantaneous channel capacity is the largest. Simulation results show that, over slow varying channels, or over Rician channels with a large  $K$  factor, opportunistic transmission scheduling can improve system performance in terms of channel capacity.

## 1. INTRODUCTION

Wireless multiple-input multiple-output (MIMO) technology can significantly increase the data rate by exploiting the extra degrees of freedom afforded by the multiple antennas at both transmitter and receiver [1, 2]. If the transmitter has perfect channel knowledge, the “water-filling” power allocation maximizes the channel capacity [3]. Otherwise, the equal power allocation strategy provides the largest channel capacity [2]. However, the large increase in capacity is often compromised by adverse channel condition [4].

In a multiuser system, different user experiences different channel which results in “multiuser diversity”. A strategy that exploits the multiuser diversity is to schedule at any time only the user with the best channel to communicate with the base station. In a practical wireless system, the multiuser diversity may be limited by slow variation of the channel or a large  $K$  factor in the Rician channel model. Viswanath *et al.* proposed a scheme to randomize the transmit powers and phases resulting in faster channel fading. This is termed *opportunistic beamforming* [5]. The transmission is scheduled to the user who is close to being in the beamforming configuration. This scheduling algorithm also maintains fairness across users.

In this paper, we propose a transmission scheduling scheme that exploits the multiuser diversity in a multiuser MIMO system. The base station uses a transmission randomizing matrix to induce rapid variation. The randomization is necessary when the channel varies slowly or has a dominant line-of-sight (LOS) component. With a common pilot, each mobile user estimates the mutual information between the transmitter and receiver. The estimated information is fed back as a measure of channel quality. The base station then “fairly” schedules transmission to the user with the best channel, assuming that it has the ability to adapt the coding schemes for various data rates as a function of the instantaneous channel capacity.

This paper is organized as follows. In Section 2, a single wireless MIMO link is discussed with transmit power allocation. In Section 3, we exploit the multiuser diversity in MIMO systems, and apply the proportional scheduling scheme to ensure fairness across users. In Section 4, we propose our opportunistic transmission scheduling scheme for a multiuser MIMO system. Section 5 gives the simulation results, and Section 6 draws the conclusion.

## 2. MIMO SYSTEMS WITH TRANSMIT POWER ALLOCATION

Consider a single-user, point-to-point wireless link with  $M_t$  transmit and  $M_r$  receive antennas as shown in Fig. 1. Assume that the channel is quasi-static and the channel response is flat over frequency. Therefore, the received signal vector  $\mathbf{y}$  can be related to the transmitted signal vector  $\mathbf{x}$  as

$$\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{x} + \mathbf{n} \quad (1)$$

where  $\mathbf{H}$  is a  $M_r \times M_t$  complex channel matrix, and  $\mathbf{n}$  is the additive white Gaussian noise vector with covariance  $N_0\mathbf{I}_{M_r}$ .  $\mathbf{W}$  is the power allocation matrix. The transmitter is constrained in its total power to  $P$  as

$$\text{tr}\{\mathbf{W}\Sigma_x\mathbf{W}^\dagger\} \leq P \quad (2)$$

where  $\Sigma_x$  is the covariance matrix of  $\mathbf{x}$ , and  $\dagger$  denotes conjugate transpose. Assuming that  $\mathbf{x}$  is i.i.d. circularly symmetric complex Gaussian, and  $\Sigma_x = (P/M_t)\mathbf{I}_{M_t}$ , we have

$$\|\mathbf{W}\|_F \leq \sqrt{M_t}P \quad (3)$$

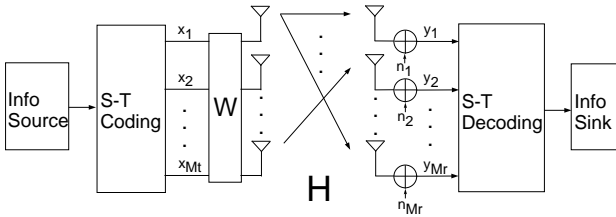


Fig. 1. Block diagram of a single wireless MIMO link.

where  $\|\circ\|_F$  denotes the Frobenius norm. The mutual information  $\mathcal{I}(\mathbf{x}; \mathbf{y})$  is given by [2]

$$\mathcal{I}(\mathbf{x}; \mathbf{y}) = \log_2 \det(\mathbf{I}_{M_r} + \frac{P}{N_0 M_t} \mathbf{H} \mathbf{W} \mathbf{W}^\dagger \mathbf{H}^\dagger) \quad (4)$$

The optimal solution that gives maximum mutual information is essentially the “water-filling” solutions for parallel Gaussian channels [3], with

$$\mathbf{W} = \mathbf{V} \mathbf{\Gamma}^{1/2} \quad (5)$$

where  $\mathbf{V}$  is taken from the singular value decomposition of the channel matrix as  $\mathbf{H} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^\dagger$ .  $\mathbf{\Gamma}$  is a diagonal matrix whose elements are calculated through the “water-filling” algorithm

$$\gamma_k = (\mu - \lambda_k^{-1})^+ \quad (6)$$

where  $\lambda_k$  are the eigenvalues of  $\mathbf{H} \mathbf{H}^\dagger$ ,  $\mu$  is chosen to meet the power constraint, and  $(a)^+$  denotes  $\max\{0, a\}$ .

In the absence of channel knowledge at the transmitter, uniform transmit power allocation across antennas maximizes the ergodic channel capacity,  $C = E\{\mathcal{I}(\mathbf{x}; \mathbf{y})\}$ . The question now is: “In the absence of channel knowledge at the transmitter, can we achieve larger downlink channel capacity by exploiting the multiuser diversity in a multiuser MIMO system?”

### 3. MULTIUSER DIVERSITY AND PROPORTIONAL FAIR SCHEDULING

In a wireless multiuser communication system, a base station communicates with multiple mobile users. Multiuser diversity offers an opportunity that at most of the time, among independently varying channels, the largest instantaneous channel capacity exceeds the ergodic channel capacity. By communicating with users with largest instantaneous channel capacity at all times, the overall spectral efficiency of the system can be improved. However, each mobile user will need to track its own instantaneous channel capacity, through, e.g. a common downlink pilot, and feedback the capacity value to the base station. The base station can serve as a scheduling agent. We consider a time-slotted system in which time is the resource to be allocated to users. The

length of a scheduling time slot depends on how fast the channels vary.

In practice, the statistics of the channel variation are not symmetrical among users. Also, there are latency constraints. A proportional fair scheduling algorithm is proposed in [5], which keeps track of the average throughput  $T_k(t)$  of each user. In time slot  $t$ , the scheduling algorithm simply transmits to user  $k^*$  with the largest  $\frac{C_k(t)}{T_k(t)}$ . The average throughputs  $T_k(t)$  can be updated with a past window of length  $t_c$  as

$$T_k(t+1) = \begin{cases} (1 - \frac{1}{t_c})T_k(t) + \frac{1}{t_c}R_k(t), & k = k^* \\ (1 - \frac{1}{t_c})T_k(t), & k \neq k^* \end{cases} \quad (7)$$

where we simply use the channel capacity  $C_k(t)$  to substitute the data rate  $R_k(t)$  in our algorithm. The parameter  $t_c$  is determined by the latency constraint of the application.

Therefore, users compete for resources based on their instantaneous channel capacities normalized by their respective average throughputs. The algorithm schedules a user when its instantaneous channel capacity is high relative to its average channel condition over the time scale  $t_c$ . As such, the benefit of multiuser diversity can be exploited. The fair scheduling strategy maximizes the total system capacity as well as the throughputs of individual users.

### 4. OPPORTUNISTIC TRANSMISSION SCHEDULING

In the case of slow channel variation where each user channel  $\mathbf{H}_k$  remains constant for a long time, or the channels have dominant LOS components, little multiuser diversity gain could be exploited. The idea here is to expedite the channel fluctuation through a transmission randomizing matrix  $\mathbf{W}$  as described in Section 2. Therefore, the instantaneous channel capacity of each user varies more, thereby providing an opportunity to exploit multiuser diversity.

To make the “composite channel”,  $\mathbf{H} \mathbf{W}$ , be in the “water-filling” configuration with high probability, the random matrix  $\mathbf{W}$  needs to be drawn from a specific distribution, which is calculated from the distribution of  $\mathbf{H}$  and the transmit power constraint. Assume that  $\mathbf{H} \in \mathcal{C}^{M_r \times M_t}$  has i.i.d. zero mean circularly symmetric complex Gaussian entries. From (5), we generate independently random matrix  $\mathbf{V}$  and random diagonal matrix  $\mathbf{\Gamma}$ .  $\mathbf{\Gamma}$  can be generated according to (6), where  $\mu$  is determined by the power constraint, and  $\{\lambda_k\}$  are the eigenvalues of  $\mathbf{H} \mathbf{H}^\dagger$  with joint probability density (unordered) given by [2]

$$p_\lambda(\lambda_1, \dots, \lambda_m) = \frac{e^{-\sum_i \lambda_i}}{m! K_{mn}} \prod_i \lambda_i^{n-m} \prod_{i < j} (\lambda_i - \lambda_j)^2 \quad (8)$$

where  $n = \max\{M_t, M_r\}$ ,  $m = \min\{M_t, M_r\}$ , and  $K_{mn}$  is a normalizing factor. The transmission randomizing ma-

trix  $\mathbf{W}$  is applied so that for any particular channel realization, there is always a good chance that the mutual information approaches its maximum value. Because the power allocation parameters  $\{\gamma_k\}$  for “water-filling” configuration are not linearly related to the eigenvalues, we may need to apply random antenna subset selection in a low SNR region [6]. The antenna subset selection in our implementation has a random fashion, in that the number and allocation of antennas in the subset are chosen randomly.

Each mobile user uses a common downlink pilot to track the quality of its “composite channel”. The channel quality is characterized by the estimated mutual information. As in Section 2, we assume that the transmitted pilot signal  $\mathbf{x}$  is composed of  $M_t$  statistically independent equal power components, each is circularly symmetric Gaussian. The mutual information can be estimated by [7]

$$\hat{\mathcal{I}}(\mathbf{x}; \mathbf{y}) = \log \frac{\det \hat{\mathbf{R}}_x \det \hat{\mathbf{R}}_y}{\det \hat{\mathbf{R}}_u} \quad (9)$$

with

$$\hat{\mathbf{R}}_x = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i \mathbf{x}_i^\dagger, \quad \hat{\mathbf{R}}_y = \frac{1}{N} \sum_{i=1}^N \mathbf{y}_i \mathbf{y}_i^\dagger,$$

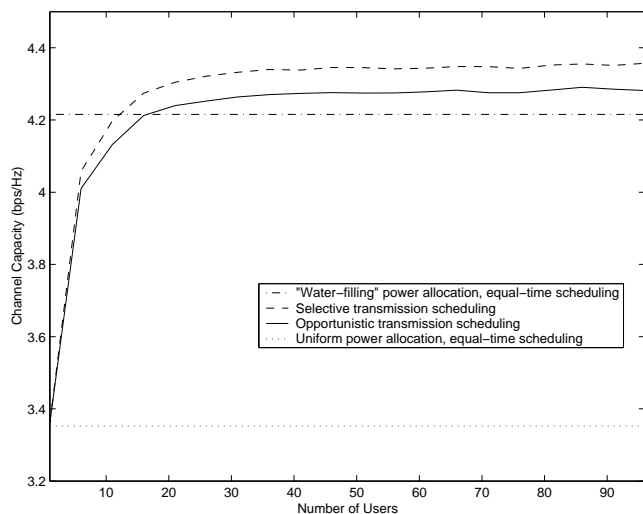
$$\hat{\mathbf{R}}_u = \frac{1}{N} \sum_{i=1}^N \mathbf{u}_i \mathbf{u}_i^\dagger.$$

where  $\mathbf{u}$  is the  $M_t + M_r$  dimensional vector  $[\mathbf{x}^T \ \mathbf{y}^T]^T$ , and  $N$  is the length of the pilot signal.

A short pilot signal is sent when the randomizing matrix  $\mathbf{W}$  changes. The variation rate of  $\mathbf{W}$  is a design parameter. It may be desirable that this rate be as fast as possible to provide a full channel fluctuation within the latency constraint. On the other hand, the variation should be slow enough so that the channel can be reliably estimated by the users. Only a small amount of feedback is needed for the the mobile users to inform the base station about their channel quality. The base station then “fairly” schedules transmission to those users that have peak capacity values, hopefully the ones closest to being in the “water-filling” configuration. In this paper, the effect of training on the capacity is not addressed. We will incorporate training overhead and channel estimation error [8] in further research.

## 5. SIMULATIONS

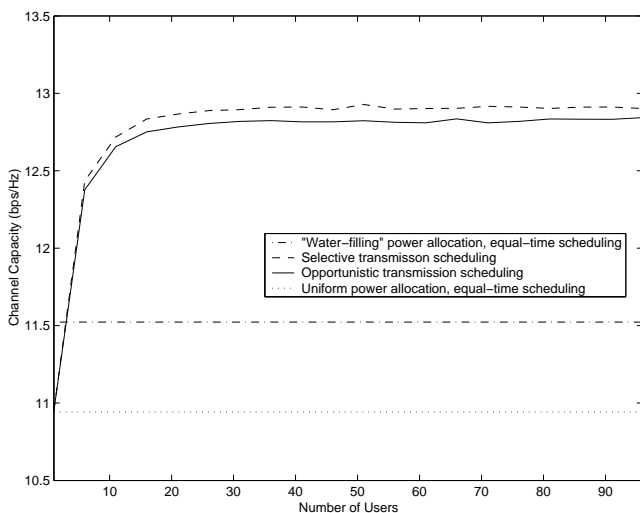
A multiuser wireless cell is simulated. The base station and each mobile user have four-element antenna array. Assume that the channels between the base station and the users have same statistics. The base station schedules different time slots to individual users, so that at any time it communicates with only one user.



**Fig. 2.** Downlink channel capacities with transmission scheduling in a fixed environment. Average receive SNR = 0 dB.

First consider the case in which all users experience slow channel variation. The entries of the  $4 \times 4$  channel matrices are i.i.d. zero mean unit variance circularly symmetric complex Gaussian. They remain constant during the entire transmission. Fig. 2 and Fig. 3 show the downlink channel capacity versus the number of mobile users in the system, with average receive SNR 0 dB and 10 dB, respectively. The transmission scheduling has latency constraint, that the average throughput tracking window has a length of  $t_c = 100 T_0$ , where  $1/T_0$  is the scheduling rate. In the selective transmission scheduling scheme, the base station transmits with uniform power allocation. In the opportunistic transmission scheduling scheme, the random matrices  $\mathbf{V}$  and  $\mathbf{\Gamma}$  of power allocation matrix  $\mathbf{W}$  are generated by first generating a random  $\mathbf{H}$ . Then in both cases, the data transmission is scheduled to the user that has a link with largest channel capacity.

The downlink channel capacities obtained by the above schemes are compared with those of two scenarios of equal-time transmission scheduling: 1). The transmitter has uniform power allocation, 2). The transmitter has full knowledge of the channel and applies “water-filling” power allocation. No multiuser diversity gain could be exploited in these two situations. Therefore, the MIMO system capacity does not change as the number of users increases. The performance of the scheduling schemes that exploit multiuser diversity, on the other hand, improves as the number of users increases. As the user number becomes large, e.g. more than 18 at low SNR, more than 4 at high SNR in the simulations, the downlink channel capacity exceeds that of the “water-filling” scenario. This is due to a large selection pool of users who may be near their perspective “water-filling” con-



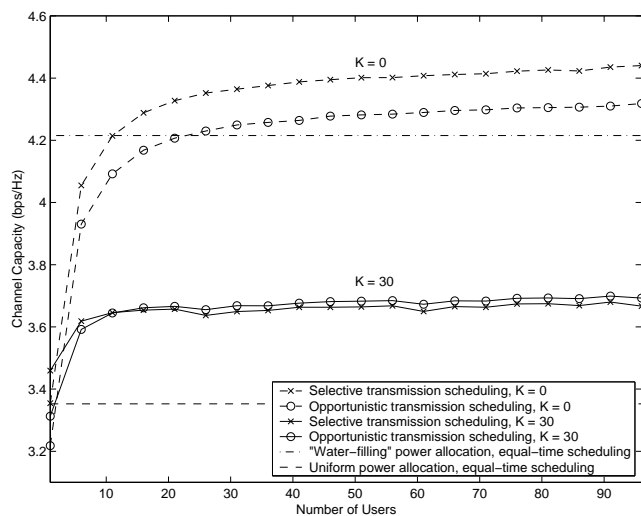
**Fig. 3.** Downlink channel capacities with transmission scheduling in a fixed environment. Average receive SNR = 10 dB.

figurations. The selective transmission scheduling slightly outperforms opportunistic transmission scheduling. This is because randomization may deteriorate the nice decorrelation property of Rayleigh channel matrix that we simulated at first place.

Fig. 4 illustrates the performance of a multiuser MIMO system where the user channels experience fast variation. Each user in the cell has an average receive SNR 0 dB. The average throughput tracking window has a length of  $t_c = 100 T_0$ , where  $1/T_0$  is the  $\mathbf{W}$  variation rate and the scheduling rate. For Rayleigh channels, i.e.  $K = 0$ , the performance of applying randomizing matrix  $\mathbf{W}$  is no better than that of uniform transmit power with selective transmission scheduling. This is because of the same reason that the modeled Rayleigh channels are near-ideal channels that demonstrate little correlation. We expect that the performance of the transmission scheduling schemes with or without pre-randomization have similar performance in practical fading channels. For Rician channels with a large  $K$  factor, e.g.  $K = 30$  in the figure, the performance improvement of the proposed opportunistic transmission scheduling over selective transmission scheduling is seen.

## 6. CONCLUSION

This paper introduced an opportunistic transmission scheduling scheme to multiuser MIMO systems. By exploiting the multiuser diversity, the fair scheduling scheme increases both system throughput and individual user throughput. This technique can be implemented with moderate complexity, and only small amount of feedback is required from the users to the base station. The transmission randomization



**Fig. 4.** Downlink channel capacity with transmission scheduling in a fast varying environment. Average receive SNR = 0 dB.

can enhance multiuser diversity gain in slowly varying channels or Rician channels. Issues for further exploration include applying this scheme to realistic correlated channels, determining the pilot length for reliable channel estimation, and incorporating the system overhead and the estimation error in capacity calculation.

## 7. REFERENCES

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