Spectral- and Energy-Efficient Transmission with Joint Bandwidth Assignment & Power Allocation

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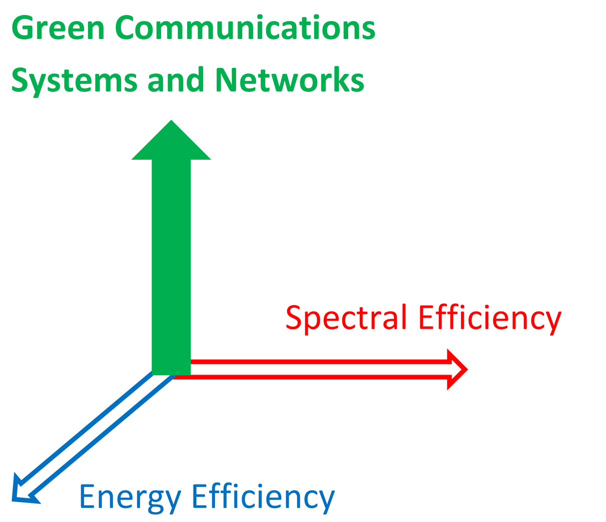
Summary

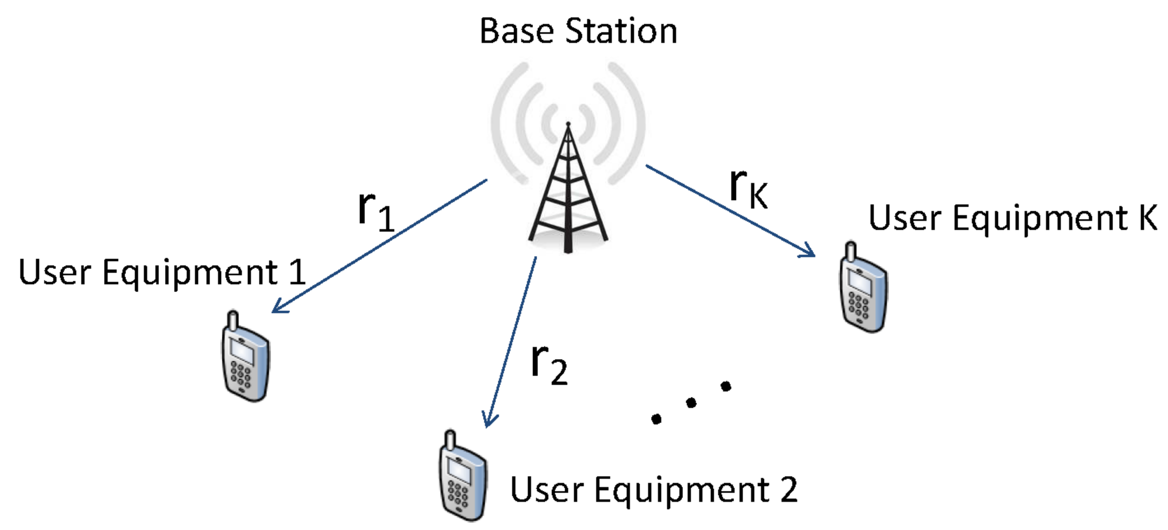
1. Spectral efficiency and energy efficiency are addressed for transmission over channels that are orthogonal in frequency. The maximum sum rate is used in measuring the spectral- and energy-efficiency.
2. Combined spectral- and energy-efficiency is maximized with *joint* bandwidth assignment and transmit power allocation.
3. With an emphasis on spectral efficiency or energy efficiency, an algorithm is provided to determine the optimal total bandwidth or the optimal total transmit power.

System Model



Spectral efficiency is measured as the comm data rate per unit bandwidth used. Energy efficiency is measured as the comm data rate per unit of power.

Frequency-orthogonal parallel broadcast:



Maximum achievable data rate ($\forall k \in \mathcal{K}$):

$$r_k = w_k \log_2\left(1 + \frac{p_k g_k}{w_k}\right)$$

Minimum rate requirement: $r_k \geq R_k, \ \forall k \in \mathcal{K}$.
Spectral Efficiency: $\Gamma_{\mathrm{SE}} \triangleq R/W$
Energy Efficiency: $\Gamma_{\mathrm{EE}} \triangleq R/P$
where, $R = \sum_{k\in\mathcal{K}} r_k$, $W = \sum_{k\in\mathcal{K}} w_k \leq W_M$, and $P = \sum_{k\in\mathcal{K}} p_k \leq P_M$.

Problem Formulation

A multi-objective optimization problem:

$$\begin{aligned} \underset{W,P,\{w_k\},\{p_k\}}{\text{maximize}} \quad & \Gamma_{\mathrm{SE}}, \Gamma_{\mathrm{EE}} \\ \text{subject to} \quad & w_k \log_2\left(1 + \frac{p_k g_k}{w_k}\right) \geq R_k, \ \ \forall k \in \mathcal{K} \\ & \textstyle\sum_{k\in\mathcal{K}} w_k = W \leq W_M \\ & \textstyle\sum_{k\in\mathcal{K}} p_k = P \leq P_M. \end{aligned}$$

It is converted into a single-objective optimization problem (with a total used bandwidth W and a total used transmit power P):

$$\begin{aligned} \underset{\{w_k\},\{p_k\}}{\text{maximize}} \quad & \Gamma_{\mathrm{EE}} + \gamma\Gamma_{\mathrm{SE}} = \left(\tfrac{1}{P} + \tfrac{\gamma}{W}\right) R(W,P) \\ \text{subject to} \quad & w_k \log_2\left(1 + \frac{p_k g_k}{w_k}\right) \geq R_k, \ \ \forall k \in \mathcal{K} \\ & \textstyle\sum_{k\in\mathcal{K}} w_k = W \leq W_M \\ & \textstyle\sum_{k\in\mathcal{K}} p_k = P \leq P_M. \end{aligned}$$

Joint Optimization Results

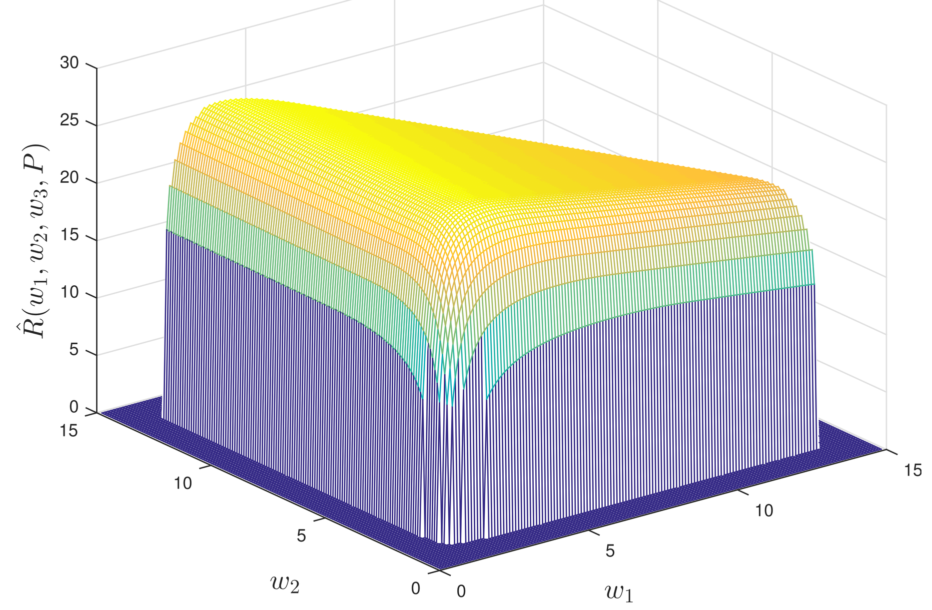


Figure 1: Maximum achievable sum rate with arbitrary bandwidth assignment, given the total bandwidth W and the total transmit power P.

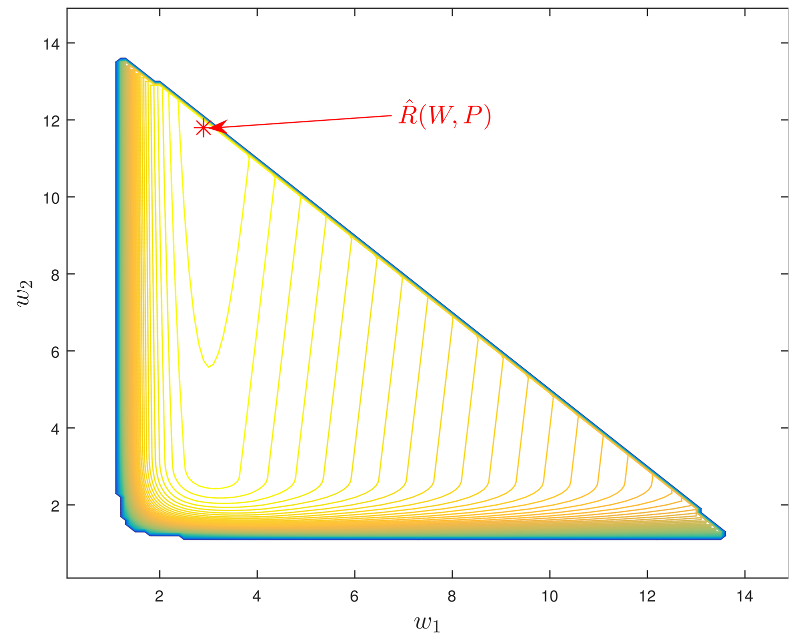


Figure 2: Contour plot. Red star indicates the peak $\hat{R}(W,P)$ with joint optimal bandwidth assignment and transmit power allocation.

Joint Optimization

Theorem: The joint bandwidth assignment and transmit power allocation problem is a convex optimization problem. The optimum sum rate $\hat{R}(W,P)$ is achieved as follows.

1. The $K-1$ UEs with channel qualities $g_i, i = 2,3,\ldots,K$, are transmitted to at their corresponding minimum required rates $R_i, i = 2,3,\ldots,K$. All of the remaining resource of the spectrum and the transmit power is used for transmission to the one UE with the best channel quality g_1.

2. The bandwidth assigned and the transmit power allocated to the ith UE, $i = 2,3,\ldots,K$, are respectively

$$w_i = \frac{R_i \ln 2}{\mathcal{W}_0(\frac{\psi g_i - 1}{e}) + 1}$$
$$p_i = \frac{R_i \ln 2}{g_i} \cdot \frac{\psi g_i - 1 - \mathcal{W}_0(\frac{\psi g_i - 1}{e})}{\mathcal{W}_0(\frac{\psi g_i - 1}{e})(\mathcal{W}_0(\frac{\psi g_i - 1}{e}) + 1)}$$

where $\mathcal{W}_0(\cdot)$ is the principal branch of the Lambert W function. The bandwidth assigned and the transmit power allocated to the UE with the best channel quality g_1 are

$$w_1 = W - \sum_{i=2}^{K} w_i \quad \text{and} \quad p_1 = P - \sum_{i=2}^{K} p_i.$$

The intermediate coefficient ψ satisfies

$$\psi = \left(\frac{1}{g_1} + \frac{p_1}{w_1}\right) \ln\left(1 + \frac{p_1 g_1}{w_1}\right) - \frac{p_1}{w_1}.$$

3. The maximum combined efficiency is

$$\left(\frac{1}{P} + \frac{\gamma}{W}\right)\hat{R}(W,P) = \left(\frac{1}{P} + \frac{\gamma}{W}\right) \cdot \left(\sum_{i=2}^{K} R_i + \frac{w_1}{\ln 2}\left(\mathcal{W}_0\left(\frac{\psi g_1 - 1}{e}\right) + 1\right)\right)$$

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Maximization of Efficiency

The maximum combined efficiency $\max(\Gamma_{\mathrm{EE}} + \gamma\Gamma_{\mathrm{SE}})$ is continuously differentiable in W and P, given that the (W,P)-pair is in the feasible set that satisfies the minimum rate requirements.
The derivative-based methods can be used to find the optimal total bandwidth that maximizes Γ_{SE} as $\gamma \to \infty$ and the optimal total transmit power that maximizes Γ_{EE} as $\gamma \to 0$.

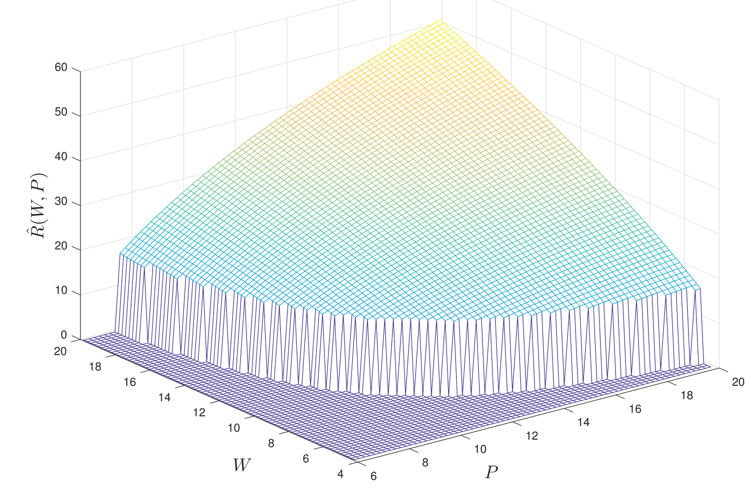

Figure 3: Maximum achievable sum rate $\hat{R}(W,P)$ with joint optimal bandwidth assignment and transmit power allocation.

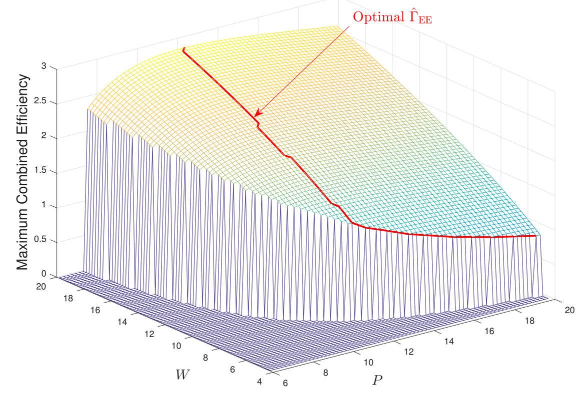

Figure 4: Maximum combined efficiency ($\approx \hat{\Gamma}_{\mathrm{EE}}$) as $\gamma = 0.01$. Solid red curve indicates the peak $\hat{\Gamma}_{\mathrm{EE}}$ with optimal total P.

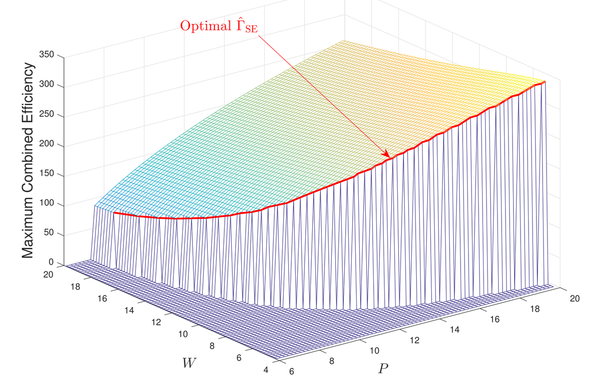

Figure 5: Maximum combined efficiency ($\approx \hat{\Gamma}_{\mathrm{SE}}$) as $\gamma = 100$. Solid red curve indicates the peak $\hat{\Gamma}_{\mathrm{SE}}$ with opt. total W.

Conclusion

Optimal bandwidth assignment and transmit power allocation are jointly derived to maximize the sum rate and the combined efficiency. The optimal solution guarantees the minimum rate requirement of each user while dedicating the excess spectrum and power resources to the one user with the best channel quality.

With an emphasis on spectral efficiency or energy efficiency, a practical method is developed to find optimal total bandwidth or optimal total transmit power that provides the best efficiency.