# FREQUENCY-DOMAIN TURBO EQUALIZATION FOR SINGLE CARRIER MOBILE BROADBAND SYSTEMS

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## ABSTRACT

*This paper investigates mobile broadband communications, which undergo time-varying radio channels with large multipath delay spread. Space-time block transmission and single carrier modulation are adopted at the transmitter to combat the inter-symbol interference resulted from the multipath delay. The single carrier modems with frequency-domain equalization have similar performance and complexity as orthogonal frequency-division multiplexing, yet less sensitive to radio frequency impairments. At the receiver, we propose the Turbo equalization that consists of a minimum mean-square error equalizer in the frequency domain and a channel decoder. In order to cope with the time-varying characteristics of the mobile channel, the received data is partitioned into blocks for the linear equalizer, and then combined as a decoding frame for the decoder. Simulation results show good performance of the proposed scheme with feasible computational complexity.*

## I. INTRODUCTION

Single carrier (SC) modulation and orthogonal frequency-division multiplexing (OFDM) are two major techniques to combat the inter-symbol interference (ISI) characterizing the dispersive channels of wireless broadband systems. Much work has been done to compare these two approaches [1], [2], [3]. Although OFDM has already been applied in many practical applications, SC modulation is gaining popularity. SC modulation uses a single carrier, instead of many sub-carriers in OFDM. Therefore, the peak-to-average ratio of the transmitted power for SC-modulated signal is smaller. This means a smaller linear range of power amplification to support a given average power, thus an inexpensive power amplifier compared with the OFDM system. When combined with frequency-domain equalization, the SC approach delivers performance similar to OFDM, with essentially the same overall complexity [4].

When multiple antennas are used at the transmitter, the space-time block transmission can be used along with SC modulation to improve system performance over frequency-selective channels [5]. Diversity transmission using Alamouti's space-time block-codes (STBC) [6] has been proposed in several wireless standards due to its many attractive features. It achieves full spatial diversity at full transmission rate for two transmit antennas, without requiring channel state information at the transmitter. At the receiver, the maximum likelihood decoding for Alamouti's STBC requires only simple linear processing. In this paper, we follow [5] to use the Alamouti-like space-time block transmission over broadband channels. The blocks are extended with cyclic prefix (CP) and transmitted with SC. Convolutional code is adopted as the forward error control method.

The receiver implements Turbo equalization. The Turbo equalization, first proposed in [7], borrowed the idea of Turbo decoding to detect iteratively the original information bits impaired by ISI. The outer code is usually a convolutional code. The ISI channel, equivalent to the inner code, is considered as a rate one convolutional code in real Galois field [8]. The extrinsic information transfer (EXIT) chart is used as a theoretical tool for performance analysis. Our proposed Turbo equalizer is composed of a linear minimum mean-square error (MMSE) equalizer in the frequency domain and a soft-input soft-output (SISO) decoder. MMSE space-frequency equalization for SC multiple-input multiple-output (MIMO) systems over frequency-selective channels is proposed in [9], but it can not be implemented iteratively to form Turbo equalization because it assumes fixed statistics about the transmitted information. To realize the Turbo equalization, we proposed a frequency-domain MMSE equalizer based on dynamic *a priori* information [10] to cooperate with the SISO outer decoder. The *a priori* information of the MMSE equalizer is updated by the SISO decoder at each iteration.

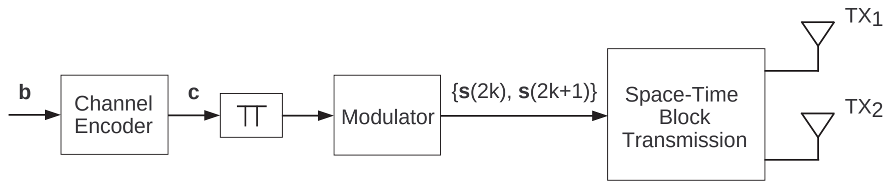


Fig. 1. Transmitter baseband block diagram: channel encoding and space-time block transmission.

Moreover, in order to cope with fast channel variation, the symbols are partitioned into short blocks for space-time block transmission. The received data are partitioned into blocks before converted to the frequency domain and fed into the linear MMSE equalizer. The equalized symbols are brought back to the time domain, and combined as a long decoding frame for the deinterleaver and the decoder. With little variation of the channel during the transmission of two blocks, the independence of the frequency tones is almost preserved. The small crosstalk between the frequency tones can be further dealt with by the frequency-domain equalization.

This paper is organized as follows. Section II describes the system model of the single carrier space-time transmission and reception. Section III details the receiver structure and scheme of the frequency-domain Turbo equalization. Numerical results through simulations are presented in Section IV, which demonstrate an improved link. Finally, conclusions are drawn in Section V.

## II. SYSTEM MODEL

### *A. Space-Time Transmission*

Fig. 1 depicts the block diagram of a wireless transmitter with two antennas. The binary bits **b** are encoded using a convolutional code. The coded bits **c** are randomly interleaved in order that the influence of error bursts is reduced at the input of the decoder at the receiver. The interleaved coded bits are then modulated and mapped to symbols and grouped into blocks of length $N$. A pair of two blocks are transmitted through the two antennas using Alamouti's space-time block transmission scheme. The block length $N$ can be adjusted to adapt to the time-varying rate of the mobile channel. We choose $N$ to be short enough such that the channel can be regarded as time-invariant during at least the transmission of two blocks. The transmission uses a single carrier frequency.

In this paper, we assume a coherent symbol-spaced receiver frontend with perfect symbol timing and describe the system with an equivalent discrete-time baseband model. Let $\mathbf{s}(2k)$ and $\mathbf{s}(2k+1)$ denote two consecutive symbol blocks with normalized power as

$$\begin{aligned}\mathbf{s}(2k) &= [s(2kN), s(2kN+1), \cdots, \\ &\quad s(2kN+N-1)]^T \qquad (1)\\ \mathbf{s}(2k+1) &= [s((2k+1)N), s((2k+1)N+1), \cdots, \\ &\quad s((2k+1)N+N-1)]^T \qquad (2)\end{aligned}$$

These two symbol blocks are transmitted through the two antennas in the following form analogous to the Alamouti space-time code,

$$\begin{bmatrix} \mathbf{s}(2k) & -\mathbf{P}\mathbf{s}^*(2k+1) \\ \mathbf{s}(2k+1) & \mathbf{P}\mathbf{s}^*(2k) \end{bmatrix} \quad \begin{matrix} \rightarrow & \text{time} \\ \downarrow & \text{space} \end{matrix} \qquad (3)$$

where $\mathbf{P}$ is a permutation matrix that is drawn from a set of permutation matrices $\{\mathbf{P}^{(n)}\}_{n=0}^{N-1}$. Each $\mathbf{P}^{(n)}$ performs a reverse cyclic shift, such that when it is applied to a $N \times 1$ vector $\mathbf{s} = [s(0), \ldots, s(N-1)]^T$, the $p$-th entry of $\mathbf{P^{(n)}s}$ is $s((N-p+n) \bmod \mathrm{N})$. For example

$$\begin{aligned}\mathbf{P}^{(0)}\mathbf{s} &= [s(N-1), s(N-2), \ldots, s(0)]^T \qquad (4)\\ \mathbf{P}^{(1)}\mathbf{s} &= [s(0), s(N-1), s(N-2), \ldots, s(1)]^T \qquad (5)\end{aligned}$$

Suppose that the transmit filter, the broadband ISI channel, and the receive filter can be represented by a discrete-time linear filter with finite-length impulse response (FIR) of length $L$. The FIR filter can be determined by the sequence $\mathbf{h}_\mu = [h_\mu(0), \ldots, h_\mu(L-1)]^T$, where $\mu = 1, 2$ indicates the transmit antenna. For multiple transmitter-receiver antenna pairs with different channel memory, $L$ is the longest filter length. As shown in Fig. 2, a cyclic prefix (CP) of length $L$ is added to each transmitted block. Therefore, the inter-block interference can be eliminated by removing the CP at the receiver. It should be noted that with the length of the CP determined by the channel filter length $L$, the shorter the original block length $N$, the more overhead is there in the transmitted frame.

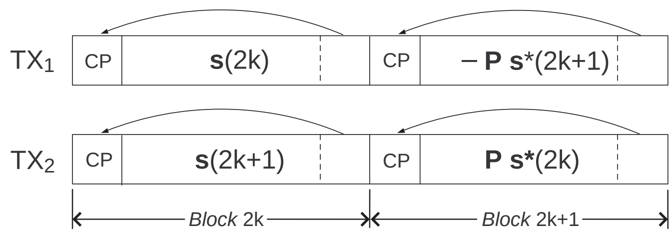


Fig. 2. Transmitted sequence through two antennas

### B. Space-Time Reception

When the receiver synchronizes to the symbol blocks, the received data in two consecutive blocks are given by

$$\mathbf{x}(2k) = \mathbf{H}_1^{(2k)}\mathbf{s}(2k) + \mathbf{H}_2^{(2k)}\mathbf{s}(2k+1) + \mathbf{n}(2k) \quad (6)$$

$$\mathbf{x}(2k+1) = -\mathbf{H}_1^{(2k+1)}\mathbf{P}\mathbf{s}^*(2k+1) + \mathbf{H}_2^{(2k+1)}\mathbf{P}\mathbf{s}^*(2k) + \mathbf{n}(2k+1) \quad (7)$$

where $\mathbf{n}$ is the zero-mean Gaussian noise vector with covariance matrix $\sigma_n^2\mathbf{I}$. With the removal of CP at the receiver, the channel $\mathbf{H}_\mu^{(j)}$ ,where $j = 2k, 2k+1$ denotes the $j$-th symbol block, can be represented by an $N \times N$ circulant matrix as

$$\mathbf{H}_\mu^{(j)} = \begin{bmatrix} h_\mu^{(j)}(0) & 0 & \cdots & h_\mu^{(j)}(1) \\ h_\mu^{(j)}(1) & h_\mu^{(j)}(0) & \cdots & h_\mu^{(j)}(2) \\ \vdots & \vdots & \ddots & \vdots \\ h_\mu^{(j)}(L-2) & h_\mu^{(j)}(L-3) & & h_\mu^{(j)}(L-1) \\ h_\mu^{(j)}(L-1) & h_\mu^{(j)}(L-2) & & 0 \\ 0 & h_\mu^{(j)}(L-1) & & 0 \\ 0 & 0 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & h_\mu^{(j)}(0) \end{bmatrix} \quad (8)$$

Here, we assume that the channel is invariant during a block transmission. Hence, $\mathbf{H}_\mu^{(j)}$ has an eigendecomposition as

$$\mathbf{H}_\mu^{(j)} = \mathbf{F}^H \mathbf{\Lambda}_\mu^{(j)} \mathbf{F} \quad (9)$$

where $\mathbf{F}$ is the orthonormal discrete Fourier transform (DFT) matrix whose $(k,l)$th entry is $\mathbf{F}_{k,l} = \frac{1}{\sqrt{N}} e^{-j(2\pi/N)kl}$ $(k,l = 0,\ldots,N-1)$. $\mathbf{\Lambda}_\mu^{(j)}$ is a diagonal matrix whose $(k,k)$th entry is given by the $k$th DFT coefficient of the first column of $\mathbf{H}_\mu^{(j)}$. In addition, the circulant matrix, when operating with the permutation matrix $\mathbf{P}$, has the following property [11]

$$\mathbf{P}\mathbf{H}_\mu^{(j)*}\mathbf{P} = \mathbf{H}_\mu^{(j)H} \quad (10)$$

As depicted in Fig. 3, the receiver divides the symbol blocks to generate $\mathbf{x}(2k)$ and $\mathbf{P}\mathbf{x}^*(2k+1)$, which are then passed through FFT modules to be converted to the frequency domain. The outputs are given by

$$\underbrace{\begin{bmatrix} \mathbf{F}\mathbf{x}(2k) \\ \mathbf{F}\mathbf{P}\mathbf{x}^*(2k+1) \end{bmatrix}}_{\mathbf{X}(2k)} = \underbrace{\begin{bmatrix} \mathbf{\Lambda}_1^{(2k)} & \mathbf{\Lambda}_2^{(2k)} \\ \mathbf{\Lambda}_2^{(2k+1)*} & -\mathbf{\Lambda}_1^{(2k+1)*} \end{bmatrix}}_{\mathbf{\Lambda}^{(2k)}} \cdot \begin{bmatrix} \mathbf{S}(2k) \\ \mathbf{S}(2k+1) \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{F}\mathbf{n}(2k) \\ \mathbf{F}\mathbf{P}\mathbf{n}^*(2k+1) \end{bmatrix}}_{\mathbf{W}(2k)} \quad (11)$$

where $\mathbf{S}(j) = \mathbf{F}\mathbf{s}(j),\ j = 2k, 2k+1$. The filtered noise $\mathbf{W}(2k)$ remains white with covariance matrix $\sigma_n^2\mathbf{I}$. The FFT output $\mathbf{X}(2k)$ is left multiplied by $\mathbf{\Lambda}^{(2k)H}$, which is a process called space-time linear combining. The combiner output is passed through the frequency-domain MMSE equalizer to obtain the estimates $\hat{\mathbf{S}}(2k)$ and $\hat{\mathbf{S}}(2k+1)$. The estimates are brought back to the time domain via IFFT modules. Finally, the coded bits are detected and decoded to the information bits. The receiver details will be explained in the next section.

## III. FREQUENCY-DOMAIN TURBO EQUALIZATION

### A. Receiver Structure

As shown in Fig. 3, the receiver consists of a space-time linear combiner, followed by the Turbo equalizer. The Turbo equalizer has two stages, the frequency-domain linear MMSE equalizer and the SISO channel decoder. The two stages are separated by a deinterleaver (a block labeled "$\pi^{-1}$") and an interleaver (a block labeled "$\pi$").

It is assumed that perfect channel estimation is available at the receiver, that is, $\mathbf{\Lambda}^{(2k)}$ is known to the receiver. With the assumption that the channel is invariant during at least the transmission time of one symbol block, $\mathbf{\Lambda}_\mu^{(j)}$ $(\mu = 1,2)$ is diagonal. With the assumption that the channel is invariant during at least the transmission time of two symbol blocks, $\mathbf{\Lambda}^{(2k)}$ is diagonal. However, when the mobile communication link undergoes fast time-varying channels, both $\mathbf{\Lambda}_\mu^{(j)}$ and $\mathbf{\Lambda}^{(2k)}$ have non-zero off-diagonal elements. That is, the multiple frequency tones at the receiver experience crosstalk. Frequency crosstalk is due to the time-selectivity of the

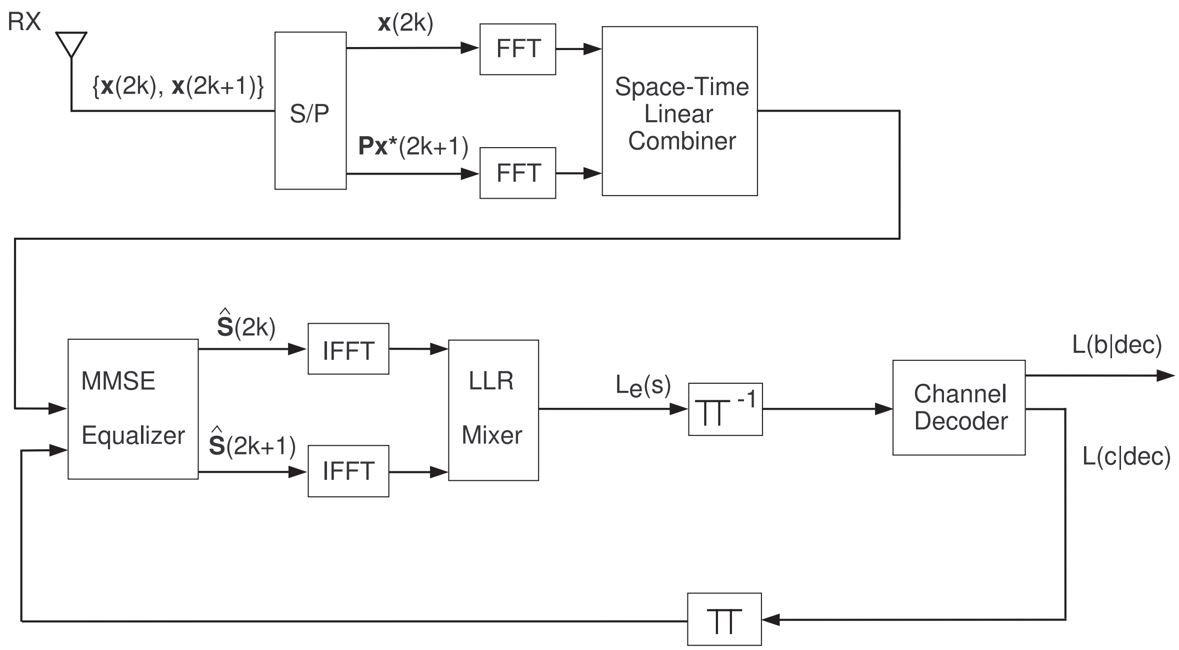


Fig. 3. Receiver baseband block diagram: space-time combining and Turbo equalization.

mobile channels. A vector linear equalizer will be used to estimate $\mathbf{S}(2k)$ and $\mathbf{S}(2k+1)$ together. The reception still benefits from the space-time block transmission and the space-time linear combining, because the off-diagonal elements are comparatively small.

For simplicity, the block index (2k) is dropped from now on, i.e. $\mathbf{X} = \mathbf{X}(2k), \boldsymbol{\Lambda} = \boldsymbol{\Lambda}^{(2k)}, \mathbf{W} = \mathbf{W}(2k)$. Let $\mathbf{S} = \begin{bmatrix} \mathbf{S}(2k) \\ \mathbf{S}(2k+1) \end{bmatrix}$, thus

$$\mathbf{S} = \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{s}(2k) \\ \mathbf{s}(2k+1) \end{bmatrix} = \tilde{\mathbf{F}}\mathbf{s}. \tag{12}$$

Following (11), we have

$$\boldsymbol{\Lambda}^H \mathbf{X} = \boldsymbol{\Lambda}^H \boldsymbol{\Lambda} \mathbf{S} + \underbrace{\boldsymbol{\Lambda}^H \mathbf{W}}_{\tilde{\mathbf{W}}} \tag{13}$$

where the filtered noise $\tilde{\mathbf{W}}$ is colored with covariance matrix as shown in (14) at the bottom of the next page.

Binary phase shift keying (BPSK) modulation is considered in this paper. The *a priori* log-likelihood ratio (LLR) of the symbol bit is fed back to the MMSE equalizer, and the equalizer outputs the *a posteriori* LLR of the symbol bit. The *a priori* LLR is given by

$$L(s(i)) = \ln \frac{P[s(i) = +1]}{P[s(i) = -1]} \tag{15}$$

The *a posteriori* LLR is given by

$$L(s(i)|\mathbf{X}) = \ln \frac{p[s(i) = +1|\mathbf{X}]}{p[s(i) = -1|\mathbf{X}]} \tag{16}$$

Using Bayes' rule, it can be written as

$$L(s(i)|\mathbf{X}) = \underbrace{\ln \frac{p[\mathbf{X}|s(i) = +1]}{p[\mathbf{X}|s(i) = -1]}}_{L_e(s(i))} + \underbrace{\ln \frac{P[s(i) = +1]}{P[s(i) = -1]}}_{L(s(i))} \tag{17}$$

where $L_e(s(i))$ is the extrinsic information.

The extrinsic information of blocks of length $N$ at the output of the MMSE equalizer is calculated and combined into a decoding frame. The decoding frame is deinterleaved to generate the *a priori* LLR of the coded bit for the channel decoder. The decoder outputs an update of the LLR of the coded bit and the information bit. The extrinsic information of the coded bits based on the code constraints is interleaved and fed back to the linear MMSE equalizer as the *a priori* information in the next iteration. The *a posteriori* LLR of the information bit is used to make hard decision at the last iteration. Note that the extrinsic information from the MMSE equalizer and from the decoder are statistically independent at the first iteration, but become more and more correlated in the subsequent iterations. Consequently, the improvement through iteration will diminish gradually.

## B. Frequency-Domain Linear MMSE Equalizer

The frequency-domain estimation $\hat{\mathbf{S}}$ at the output of the linear MMSE equalizer is given by

$$\begin{aligned}\hat{\mathbf{S}} &= [\mathbf{C}_{\mathbf{XX}}^{-1}\mathbf{C}_{\mathbf{XS}}]^H(\mathbf{X}-E(\mathbf{X}))+E(\mathbf{S}) \\ &= [(\mathbf{\Lambda}\mathbf{C}_{\mathbf{SS}}\mathbf{\Lambda}^H+\sigma_n^2\mathbf{I})^{-1}\mathbf{\Lambda}\mathbf{C}_{\mathbf{SS}}]^H(\mathbf{X}-\mathbf{\Lambda}E(\mathbf{S})) \\ &\quad +E(\mathbf{S}) \qquad (18)\end{aligned}$$

where $\mathbf{C}_{\mathbf{XX}}, \mathbf{C}_{\mathbf{XS}}$ and $\mathbf{C}_{\mathbf{SS}}$ are covariance matrices, defined as $\mathbf{C}_{\mathbf{xy}} \equiv Cov(\mathbf{x},\mathbf{y}) = E(\mathbf{x}\mathbf{y}^H) - E(\mathbf{x})E(\mathbf{y}^H)$. According to (12), $E(\mathbf{S}) = \tilde{\mathbf{F}}E(\mathbf{s})$ with the $i$th element $E(s(i))$ depending on the *a priori* LLR $L(s(i))$ as

$$\begin{aligned}E(s(i)) &= \sum_{k\in\{+1,-1\}} k\cdot P[s(i)=k] \\ &= \tanh\left(\frac{1}{2}\ln\frac{P[s(i)=+1]}{P[s(i)=-1]}\right) \\ &= \tanh\left(\frac{L(s(i))}{2}\right) \qquad (19)\end{aligned}$$

Due to the independence of the interleaved symbols $\{s(i)\}$ and the BPSK modulation, the covariance matrix $\mathbf{C}_{\mathbf{SS}}$ can be calculated as

$$\begin{aligned}\mathbf{C}_{\mathbf{SS}} &= \tilde{\mathbf{F}}\mathbf{C}_{\mathbf{ss}}\tilde{\mathbf{F}}^H \\ &= \tilde{\mathbf{F}}diag\left((1-|E(\mathbf{s})|^2)\right)\tilde{\mathbf{F}}^H \qquad (20)\end{aligned}$$

where the $i$th element of $|E(\mathbf{s})|^2$ is $|E(s(i))|^2$, given in (19). The *a priori* LLR $L(s(i))$ is updated in each iteration, so the MMSE estimator must be recalculated. Once the symbol values in the frequency domain are estimated, the time-domain values can be obtained by performing the inverse FFT as

$$\hat{\mathbf{s}} = \tilde{\mathbf{F}}^H\hat{\mathbf{S}} \qquad (21)$$

At the first Turbo iteration, it is assumed that $s(i)$ is equally likely $+1$ or $-1$ which yields $E(\mathbf{S}) = \tilde{\mathbf{F}}E(\mathbf{s}) = \mathbf{0}$ and $\mathbf{C}_{\mathbf{SS}} = \mathbf{I}$. The MMSE estimator as shown in (18) can be simplified as

$$\hat{\mathbf{S}} = [(\mathbf{\Lambda}\mathbf{\Lambda}^H+\sigma_n^2\mathbf{I})^{-1}\mathbf{\Lambda}]^H\mathbf{X} \qquad (22)$$

Assume that the probability density function $p[\hat{s}(i)|s(i)=k], k=\pm1$ is Gaussian with mean $\mu_{i,k} = E\{\hat{s}(i)|s(i)=k\}$ and variance $\sigma_{i,k}^2 = \mathrm{cov}\{\hat{s}(i),\hat{s}(i)|s(i)=k\}$. The *a posteriori* LLR $L(s(i)|\hat{s}(i))$, which is an approximation of $L(s(i)|\mathbf{X})$, can be expressed as

$$L(s(i)|\hat{s}(i)) = \underbrace{\ln\frac{p[\hat{s}(i)|s(i)=+1]}{p[\hat{s}(i)|s(i)=-1]}}_{L_e(s(i))} + \underbrace{\ln\frac{P[s(i)=+1]}{P[s(i)=-1]}}_{L(s(i))} \qquad (23)$$

The extrinsic information $L_e(s(i))$ can be calculated as

$$\begin{aligned}L_e(s(i)) &= \ln\frac{\exp(-|\hat{s}(i)-\mu_{i,+1}|^2/\sigma_{i,+1}^2)}{\exp(-|\hat{s}(i)-\mu_{i,-1}|^2/\sigma_{i,-1}^2)} \\ &= \frac{|\hat{s}(i)-\mu_{i,-1}|^2}{\sigma_{i,-1}^2} - \frac{|\hat{s}(i)-\mu_{i,+1}|^2}{\sigma_{i,+1}^2} \qquad (24)\end{aligned}$$

Let $\mathbf{A} = [(\mathbf{\Lambda}\mathbf{C}_{\mathbf{SS}}\mathbf{\Lambda}^H+\sigma_n^2\mathbf{I})^{-1}\mathbf{\Lambda}\mathbf{C}_{\mathbf{SS}}]^H$. From (18) and (21), we have

$$\hat{s}(i) = \mathbf{u}_i\tilde{\mathbf{F}}^H\mathbf{A}(\mathbf{\Lambda}\mathbf{S}-\mathbf{\Lambda}E(\mathbf{S})+\mathbf{W})+E(s(i)) \qquad (25)$$

where $\mathbf{u}_i = [0,\ldots,0,\underbrace{1}_{i\text{th}},0,\ldots,0]$. It can be derived that

$$\mu_{i,k} = \mathbf{u}_i\tilde{\mathbf{F}}^H\mathbf{A}\mathbf{\Lambda}\tilde{\mathbf{F}}\mathbf{d}^{(i,k)} + E(s(i)) \qquad (26)$$

$$\sigma_{i,k}^2 = \mathbf{u}_i\left(\tilde{\mathbf{F}}^H\mathbf{A}\left(\mathbf{\Lambda}\tilde{\mathbf{F}}\mathbf{C}_{\mathbf{ss}|s(i)=k}\tilde{\mathbf{F}}^H\mathbf{\Lambda}^H+{\sigma_n}^2\mathbf{I}\right)\mathbf{A}^H\tilde{\mathbf{F}}\right){\mathbf{u}_i}^H \qquad (27)$$

where

$$\mathbf{d}^{(i,k)} = [0,\ldots,0,\underbrace{k-E(s(i))}_{i\text{th}},0,\ldots,0]^T$$

and

$$\begin{aligned}&\mathbf{C}_{\mathbf{ss}|s(i)=k} \\ &= Cov(\mathbf{s},\mathbf{s}|s(i)=k) \\ &= \mathbf{C}_{\mathbf{ss}} + diag(0,\ldots,0,\underbrace{2E(s(i))(E(s(i))-k)}_{i\text{th}}, \\ &\quad 0,\ldots,0).\end{aligned}$$

At the first Turbo iteration, (26) and (27) can be simplified as

$$\begin{aligned}\mu_{i,k} &= k\mathbf{u}_i\tilde{\mathbf{F}}^H\mathbf{A}\mathbf{\Lambda}\tilde{\mathbf{F}}{\mathbf{u}_i}^H \qquad (28) \\ \sigma_{i,k}^2 &= \mathbf{u}_i\left(\tilde{\mathbf{F}}^H\mathbf{A}\left(\mathbf{\Lambda}\mathbf{\Lambda}^H+{\sigma_n}^2\mathbf{I}\right)\mathbf{A}^H\tilde{\mathbf{F}}\right){\mathbf{u}_i}^H \qquad (29)\end{aligned}$$

$$\mathbf{C}_{\tilde{\mathbf{W}}} = \begin{bmatrix}\mathbf{\Lambda}_1^{(2k)H}\mathbf{\Lambda}_1^{(2k)}+\mathbf{\Lambda}_2^{(2k+1)}\mathbf{\Lambda}_2^{(2k+1)H} & \mathbf{\Lambda}_1^{(2k)H}\mathbf{\Lambda}_2^{(2k)}-\mathbf{\Lambda}_2^{(2k+1)}\mathbf{\Lambda}_1^{(2k+1)H} \\ \mathbf{\Lambda}_2^{(2k)H}\mathbf{\Lambda}_1^{(2k)}-\mathbf{\Lambda}_1^{(2k+1)}\mathbf{\Lambda}_2^{(2k+1)H} & \mathbf{\Lambda}_2^{(2k)H}\mathbf{\Lambda}_2^{(2k)}+\mathbf{\Lambda}_1^{(2k+1)}\mathbf{\Lambda}_1^{(2k+1)H}\end{bmatrix}\sigma_n^2 \qquad (14)$$

### C. SISO Channel Decoder

The block length $N$ is chosen relatively small such that the channel can be assumed time-invariant during the transmission of two blocks. Also, the short block length eases the computation of the MMSE equalizer. The *a priori* LLRs of the coded bits of multiple blocks are combined as a decoding frame. The SISO channel decoder takes in the entire frame and outputs the updated LLRs of the coded bits $L(c(i)|\text{decoding})$, as well as the LLRs of the information bits $L(b(i)|\text{decoding})$ based upon the code constraints.

We use the log maximum *a posteriori* probability (Log-MAP) decoding algorithm to calculate the *a posteriori* LLRs of the coded bits and the information bits. The *a posteriori* LLRs of the coded bits can be written as [12]

$$\begin{aligned} & L(c(i)|\text{decoding}) \\ = & \ \ln \frac{P[c(i)=+1|\text{decoding}]}{P[c(i)=-1|\text{decoding}]} \\ = & \ \ln \frac{\sum_{(s',s)\in\Sigma_l^+} p[s_l=s', s_{l+1}=s]}{\sum_{(s',s)\in\Sigma_l^-} p[s_l=s', s_{l+1}=s]} \end{aligned} \tag{30}$$

while the *a posteriori* LLRs of the information bits can be written as

$$\begin{aligned} & L(b(i)|\text{decoding}) \\ = & \ \ln \frac{P[b(i)=+1|\text{decoding}]}{P[b(i)=-1|\text{decoding}]} \\ = & \ \ln \frac{\sum_{(s',s)\in U_l^+} p[s_l=s', s_{l+1}=s]}{\sum_{(s',s)\in U_l^-} p[s_l=s', s_{l+1}=s]} \end{aligned} \tag{31}$$

where $\Sigma_l^{+1}$ and $\Sigma_l^{-1}$ are the sets of all state pairs $s_l = s'$ and $s_{l+1} = s$ that correspond to the coded bit $c(i) = +1$ and $c(i) = -1$, respectively. $U_l^{+1}$ and $U_l^{-1}$ are the sets of all state pairs $s_l = s'$ and $s_{l+1} = s$ that correspond to the information bit $b(i) = +1$ and $b(i) = -1$, respectively.

## IV. SIMULATIONS

The performance of the proposed scheme is evaluated through simulations. As mentioned in Sections II-A and III-C, in order to cope with the time-varying characteristics of the mobile channel, we partition a long decoding frame into multiple blocks of length $N$ for the space-time transmitter and the equalizer. The equalized blocks are combined as input to the channel decoder. This partition can also improve the computational efficiency. Specifically in our simulations,

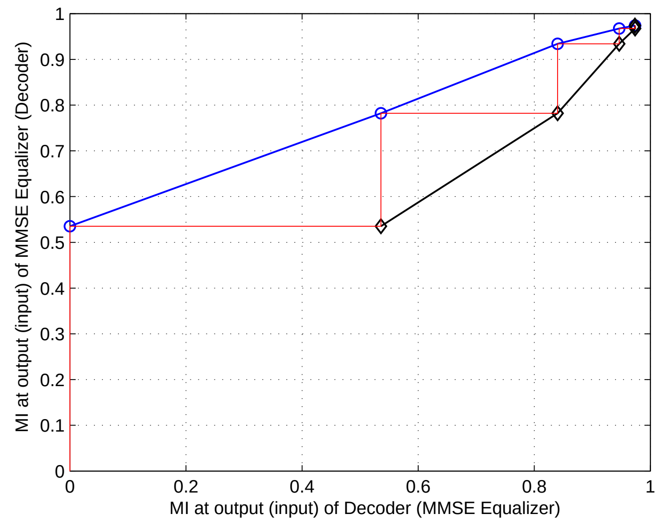


Fig. 4. EXIT chart, $E_b/N_0 = 0.8$ dB.

$2^{16}$ BPSK-modulated symbols are partitioned into 256 pairs of blocks (total 512 blocks). Each block, that contains $N = 128$ symbols, is processed through the linear frequency-domain MMSE equalizer and the outputs construct a $2^{16}$-symbol frame to be fed into the deinterleaver and the decoder. Convolutional codes and the Log-MAP decoding algorithm are used.

Fig. 4 shows the EXIT chart at $E_b/N_0 = 0.8$ dB, where $E_b$ is the energy of the information bit. The coding gain and the antenna gain are considered. The EXIT chart tracks the mutual information between an information bit and its *a posteriori* extrinsic LLR at the outputs of the equalizer and the decoder. Fig. 5 depicts the BER performance versus $E_b/N_0$ at multiple iterations. As shown in these figures, the mutual information increases with each iteration, which indicates more accurate decoding results. Consequently, the BER becomes smaller with each iteration.

The simulated system has two transmit antennas and one receive antenna, and is labeled as $2 \times 1$. The multi-antenna channel is exploited through the Alamouti-like space-time block transmission. The proposed scheme is designed to handle ISI encountered in broadband systems. Specifically in our simulation, the length of the channel impulse response is $L = 16$ symbol periods. For comparison, we also consider the case of the frequency-selective channel with a single transmit antenna, labeled $1 \times 1$ ISI channel, and the case of the frequency-flat channel with two transmit antennas, labeled $2\times1$ non-ISI channel. The comparison of the BER performance at the fourth Turbo iteration is shown in Fig. 6. It is revealed that the performance of our pro-

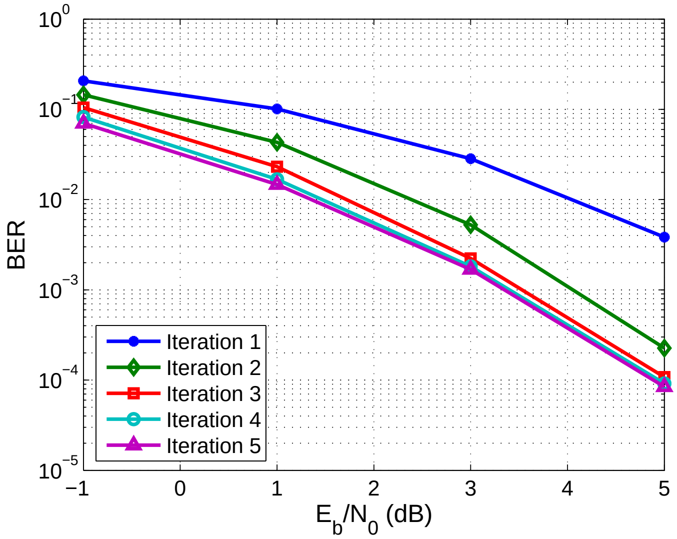


Fig. 5. Bit error rate at different iterations.

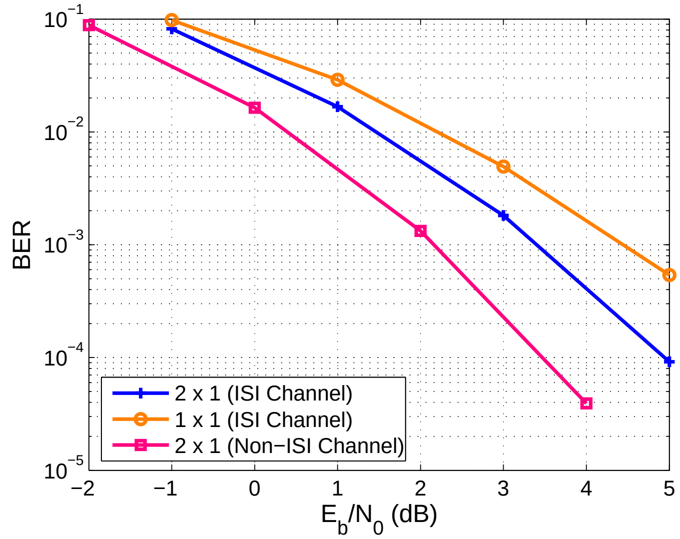


Fig. 6. Bit error rate comparison at the 4th iteration.

posed scheme over the $2 \times 1$ ISI channel is about 1.2 dB away from that of the $2 \times 1$ non-ISI channel at a BER of $10^{-3}$. Over ISI channels, better performance is achieved in the $2 \times 1$ case than in the $1 \times 1$ case. This results from the diversity gain obtained through space-time block transmission.

## V. CONCLUSION

A frequency-domain Turbo equalization scheme is proposed for space-time block transmission over SC broadband channels. The equalization is implemented through the *a priori* information based linear MMSE detector, whose transformation matrix is updated in each iteration according to the feedback from the SISO decoder. Simulation results show that the proposed scheme is promising to combat the ISI. Better performance with two transmit antennas can be achieved due to the diversity gain resulted from the space-time block transmission. Furthermore, by partitioning the long decoding frame into short blocks, the proposed approach is able to deal with time-varying mobile channels, given that the channel varies slightly during the transmission of two blocks.

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