ELC 4351: Digital Signal Processing

Leon (Liang) Dong

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Leon (Liang) Dong

Department of Electrical and Computer Engineering
Baylor University

liang_dong@baylor.edu

Quantization Errors

Errors in Computing Systems:

- Numbers are represented by a finite number of bits. The resulting errors are called the finite-wordlength or finite-precision effects.
- Quantization errors:
  Signal quantization
  Coefficient quantization
- Arithmetic errors:
  Roundoff or truncation
  Overflow

Signal Quantization

- Analog signal $x(t) \Rightarrow$ ADC $\Rightarrow$ digital signal $x[n]$.
- First, $x(t)$ is sampled and becomes a discrete-time signal $x(nT)$.
- Then, $x(nT)$ is encoded using B bits and becomes a digital signal $x[n]$.

Signal Quantization

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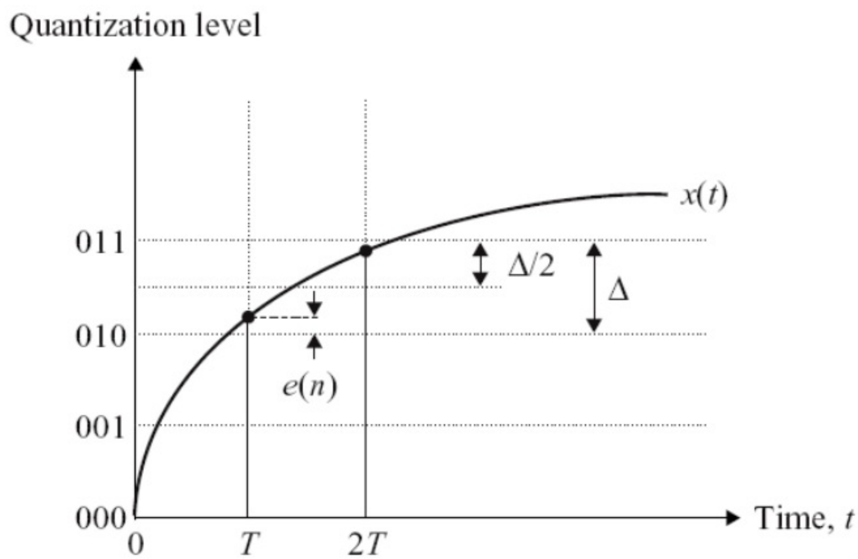
Signal to Quantization Noise Ratio
Coefficient Quantization
Roundoff Noise
Overflow
Scaling of Signals

- Suppose that $-1 \leq x[n] < 1$.
- Dynamic range $= 2$.
- B bits represent a sample, the number of quantization levels is 2^B.
- The quantization step (resolution): $\Delta = \frac{2}{2^B} = 2^{-B+1}$.

Rounding for Quantization

A 3-bit ADC:



Rounding Error

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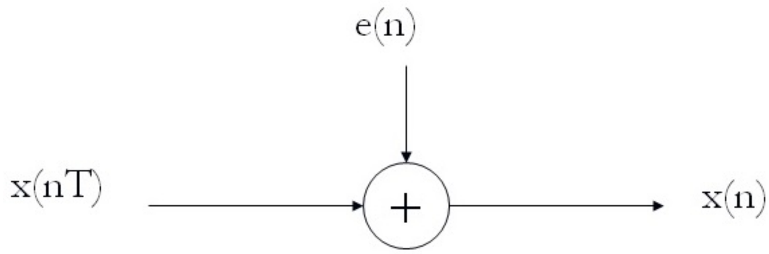
- Quantization error/noise: $e(n) = x(n) - x(nT)$.
- Rounding: $|e(n)| \leq \Delta/2$.
- The quantization noise depends on the quantization step.
- More bits $\Rightarrow$ smaller quantization step $\Rightarrow$ lower quantization noise.

Linear Model

- The nonlinear operation of quantizer: $x(n) = Q[x(nT)]$
- Linear operation: $x(n) = Q[x(nT)] = x(nT) + e(n)$



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Common Assumptions

- Assume that the quantization error $e(n)$ is uncorrelated with $x(n)$.
- Assume $e(n)$ is a random variable uniformly distributed in the interval $[-\Delta/2, \Delta/2]$.
- Therefore, $E[e(n)] = (-\Delta/2 + \Delta/2)/2 = 0$;

  and variance: $\sigma_e^2 = \frac{\Delta^2}{12} = \frac{2^{-2B}}{3}$.

  Large wordlength B leads to small quantization error σ_e^2.

Signal to Quantization Noise Ratio

- $\mathrm{SNR} = 10\log_{10}(\sigma_x^2/\sigma_e^2)$.
- With $\sigma_e^2 = 2^{-2B}/3$, we have

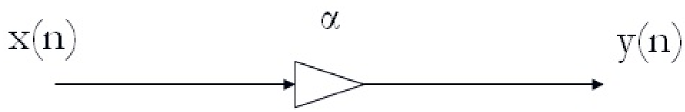
$$\begin{aligned}\mathrm{SNR} &= 10\log_{10}(3\times 2^{2B}\sigma_x^2)\\ &= 10\log_{10}3 + 20B\log_{10}2 + 10\log_{10}\sigma_x^2\\ &= 4.77 + 6.02B + 10\log_{10}\sigma_x^2\end{aligned}$$

- For each additional bit, the ADC provides about 6-dB gain.
- SNR is proportional to σ_x^2. Keep signal power as large as possible.

Coefficient Quantization

- The filter coefficients b_n, a_m are quantized for a given fixed-point processor.
- Coefficient quantization can cause serious problems if the poles of designed IIR filters are too close to the unit circle.
- This is because those poles may move outside the unit circle due to coefficient quantization, resulting in an unstable implementation.

Roundoff Noise



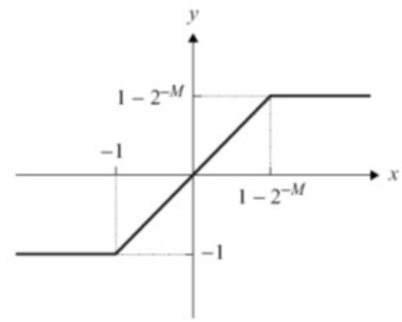
- $y(n) = \alpha x(n)$
- $x(n)$ and α are B-bit, the product $y(n)$ will be $2B$-bit.
- Usually, the result will be stored in B-bit memory.
- Truncation or rounding brings the roundoff noise.
- $y(n) = Q[\alpha x(n)] = \alpha x(n) + e(n)$
- Is this noise larger?

Overflow

- When the dynamic range of signals is fixed, the result of an arithmetic addition may exceed the capacity of the register.
- This overflow results in severe distortion of the signal output.
- We need saturation algorithm or proper scaling.

Saturation Algorithm

- Saturation arithmetic prevents overflow by keeping the result at a maximum value.
- Saturation algorithm is a nonlinear operation that clips the desired waveform.



$$y = \begin{cases} 1-2^{-M}, & x \geq 1-2^{-M} \\ x, & -1 \leq x < 1 \\ -1, & x < -1 \end{cases}$$

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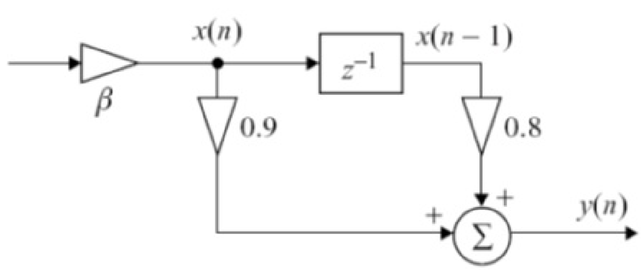
Scaling of Signals

- An effective technique in preventing overflow is by scaling down the signal.



- If the signal $x(n)$ is scaled by β, the corresponding signal variance changes to $\beta^2\sigma_x^2$.

Scaling of Signals

- $$\begin{aligned} \mathrm{SNR} &= 10\log_{10}(\beta^2\sigma_x^2/\sigma_e^2) \\ &= 4.77 + 6.02B + 10\log_{10}\sigma_x^2 + 20\log_{10}\beta \end{aligned}$$
- For down scaling, $\beta < 1$.
- The term $20\log_{10}\beta$ is negative, and the SNR reduces.
- For example, when $\beta = 0.5$, $20\log_{10}\beta = -6.02$ dB, thus reducing the SNR of the input signal by about 6 dB.
- This is equivalent to losing 1 bit in representing the signal. Why?