Reflection Coefficient and Transmission Lines Using the Smith Chart

As we discussed in class, the Smith Chart represents the complex plane of the reflection coefficient. You will recall from class that the input reflection coefficient to a transmission line of physical length l, Γ_{in}, is given in terms of the load reflection coefficient Γ_L by the expression

$$\Gamma_{in} = \Gamma_L e^{-j2\beta l} \tag{1}$$

This indicates that on the complex reflection coefficient plane (the Smith Chart), the point representing Γ_{in} can be found by a constant-radius rotation from the point representing Γ_L. This rotation represents a change in phase of the complex number, and is a rotation at constant radius because the magnitude of the reflection coefficient remains constant in (1) when finding Γ_{in} from Γ_L (please note this by careful examination of (1)). The phase changes by $-2\beta l$. This represents a rotation in the clockwise direction in the complex Γ plane (the Smith Chart) by $-2\beta l$ radians.

To graphically find Γ_{in} from Γ_L on the Smith Chart, locate Γ_L (or z_L) on the Smith Chart. Then, using your compass, draw a constant-radius circle centered at the center of the Smith Chart and going through Γ_L. Phase change of the reflection coefficient due to a transmission line will cause the value of reflection coefficient (and impedance) to rotate along this circle in the clockwise direction. The distance of rotation has been computed for you in the creation of the Smith Chart and is tabulated on the scale labeled "Wavelengths toward Generator" around the outside circumference of the Chart.

We now work through an example in the Pozar book for practice and demonstration of how these techniques work. Please pull out a Smith Chart, pencil, ruler, and compass, and work through this problem along with this tutorial.

Pozar Example 2.2, p. 66:

"A load impedance of $40 + j70$ Ω terminates a 100 Ω transmission line that is 0.3λ long. Find the reflection coefficient at the load, the reflection coefficient at the input to the line, the input impedance, the standing wave ratio on the line, and the return loss." We will leave it to Pozar to explain standing wave ratio and return loss for now. This tutorial will focus on finding the load reflection coefficient, input reflection coefficient and input impedance.

Solution:

The first step is to normalize the impedance:

$$z_L = \frac{Z_L}{Z_0} = \frac{40 + j70}{100} = 0.4 + j0.7$$

This number can be plotted on the Smith Chart. See the location marked "1" on the Chart on the attached scanned pages of my solution.

To measure Γ_L, stretch the compass to place the pin on the center of the chart and the lead of the other arm on the point representing z_L (and of course this is also Γ_L). Then go to the bottom of the page and use the scale labeled "RFL COEFF, E or I". Place the pin on the location marked "CENTER" of this scale, and make a mark with your compass' lead. Read the value associated with the mark. This is the magnitude of the reflection coefficient. This scale is nicely laid out for you to use for this purpose. This is labeled "2" on my Smith Chart attached to this document and is near the bottom of the Smith Chart page. It can be seen from the scale that the magnitude of the reflection coefficient, $|\Gamma_L|$ is approximately 0.58.

To find Γ_{in} based on this, draw a line from the center of the Smith Chart (using your ruler) through the edge of the chart. Read the reading on the outermost circumferential scale (this is the "WAVELENGTHS TOWARD GENERATOR" scale. You should read approximately 0.105. Write this on your chart. This is your starting point on this scale. I have labeled this step as "3". Now, add the length of the line, in wavelengths, to this number 0.105. This gives the number on the "WAVELENGTHS TOWARD GENERATOR" scale that should be read when you draw a line through Γ_{in}. Since in this example the line is 0.3 wavelengths long, the addition to be performed is $0.105\lambda + 0.3\lambda$ $= 0.405\lambda$. Now draw a line from the center of the Smith Chart through 0.405 on the "WAVELENGTHS TOWARD GENERATOR" scale. This is labeled as "4" on my Smith Chart. The intersection of this line and the circle you have drawn on the chart is the point representing Γ_{in} and z_{in}. The value of Γ_{in} is now easily determined. The magnitude is the same as $|\Gamma_L|$ because the point is on a constant radius circle in the reflection-coefficient plane (on the same radius as Γ_L). The angle has changed, but can be easily read by the intersection of the line through Γ_{in} with the "ANGLE OF REFLECTION COEFFICIENT IN DEGREES" scale. This is the third scale from the outside on the outer circumference of the Smith Chart. It appears that the angle of the input reflection coefficient is -112° by reading this scale. This reading is shown and is labeled as "5" on the Smith Chart.

The input impedance z_{in} is represented by the same point as Γ_{in}, because the Smith Chart is a superimposition of the resistance and reactance circles on the complex reflection-coefficient plane. Thus we can read z_{in} by finding the resistance and reactance circles that go through this point. The resistance circle appears to be 0.37 and the reactance circle appears to be -0.61 (recall that reactance circles on the bottom half of the Smith Chart represent *negative* reactances). Thus, the value of input impedance is

$$z_{in} = 0.37 - j0.61$$

The input impedance in Ohms, Z_{in}, is found by un-normalizing z_{in} (multiplying it by the reference impedance Z_0:
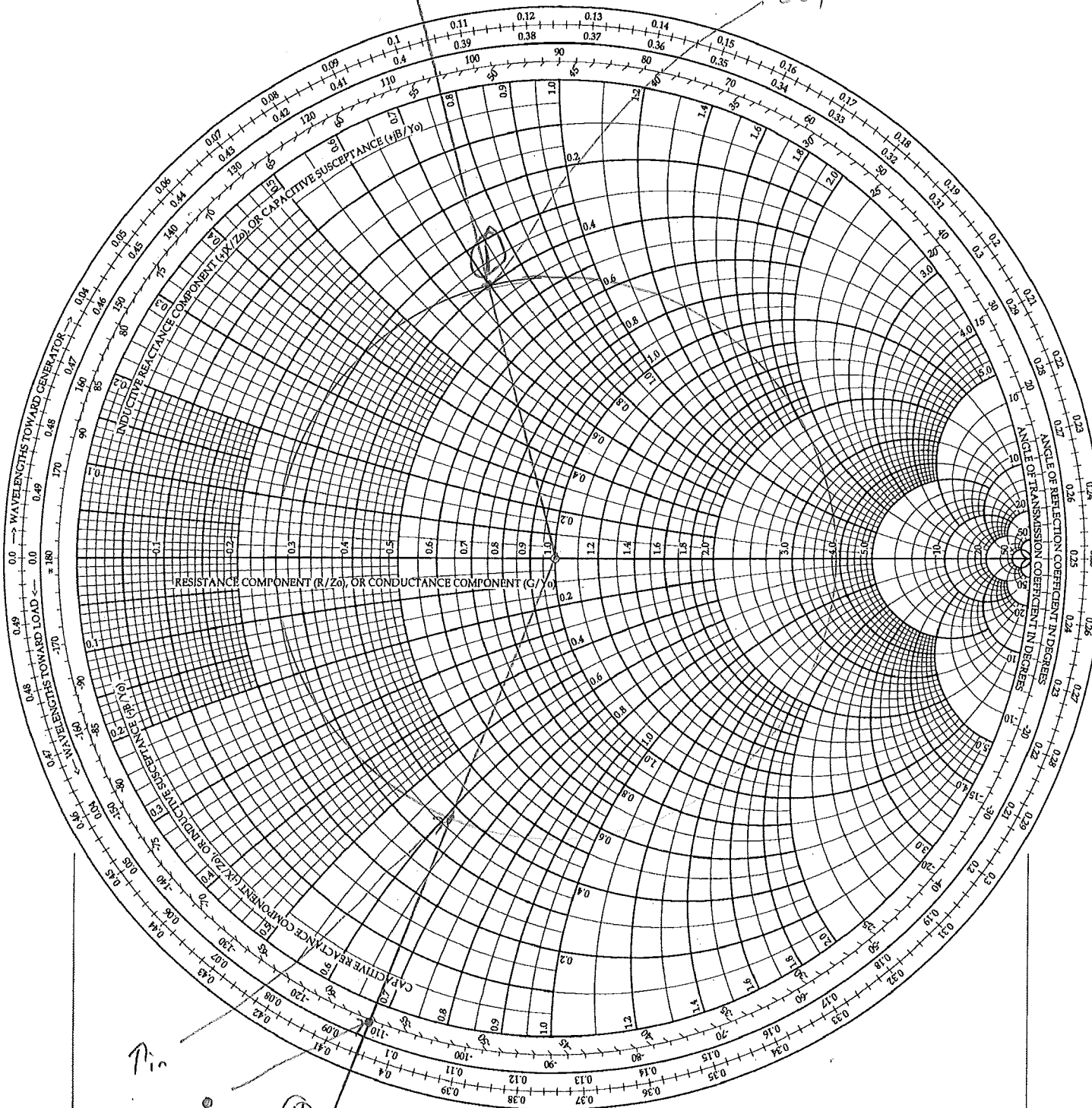
$$Z_{in} = z_{in}Z_0 = (0.37 - j0.61)(100) = (37 - j61)\ \Omega$$

Your upcoming lab (Lab 2) shows results of putting impedances on the end of a transmission line. When a load impedance is placed on a transmission line, clockwise motion with constant radius results on the Smith Chart as a result of this line being placed between the reference plane (the viewpoint) and the load impedance, as we have seen in this example. How far the rotation goes around the Smith Chart in the clock is based on the number of wavelengths long the line is (the "WAVELENGTHS TOWARD GENERATOR" scale). Note that the wavelength is a function of the frequency. At higher frequencies, the wavelength is smaller, so a physical transmission line of fixed *physical* length *l* will have a longer *electrical length βl* and will be longer in terms of the number of wavelengths it causes rotation. Thus, for higher frequencies, the rotation is greater and for lower frequencies, the rotation is not as far. In the laboratory, you will use the network analyzer to measure the input reflection coefficient of a short circuit on the end of a transmission line over a range of frequencies (300 kHz to 3 GHz). At 300 kHz, the line's length in wavelengths will be very small, but at 3 GHz, the line may be much longer. As a result, the trace of input reflection coefficient will begin near the short circuit position on the left of the Smith chart and trace clockwise at constant radius (in this case, around the edge of the Smith Chart) as frequency goes higher. If the line is physically longer, than at each frequency the line is electrically longer, and the trace will extend further around the Smith Chart. For longer lines, you may actually see over one complete rotation around the chart, indicating that the line is over half a wavelength long at the higher frequencies. (You will do the same thing for the open circuit, which begins at the open-circuit position on the right side of the Smith Chart).
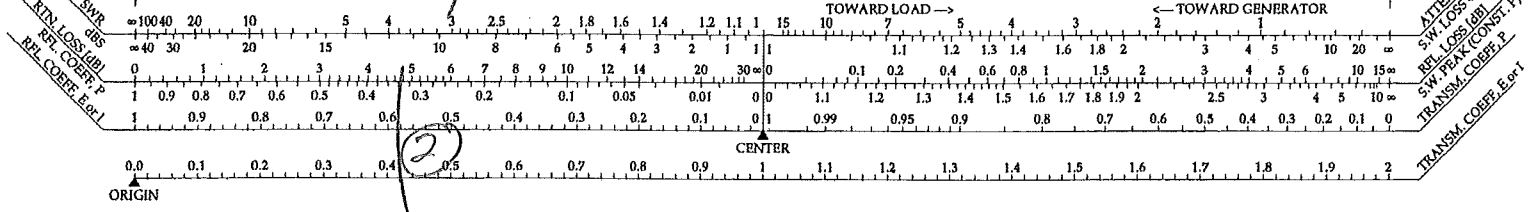
Smith Chart

0.105λ

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zι, Πₗ

RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)

Πᵢₙ

⑤ −112°

④ 0.405λ

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