

Table 4.1
A Short Table of Fourier Transforms

| | $f(t)$ | $F(\omega)$ | |
|----|---|--|-----------------------------|
| 1 | $e^{-at}u(t)$ | $\frac{1}{a + j\omega}$ | $a > 0$ |
| 2 | $e^{at}u(-t)$ | $\frac{1}{a - j\omega}$ | $a > 0$ |
| 3 | $e^{-a t }$ | $\frac{2a}{a^2 + \omega^2}$ | $a > 0$ |
| 4 | $te^{-at}u(t)$ | $\frac{1}{(a + j\omega)^2}$ | $a > 0$ |
| 5 | $t^n e^{-at}u(t)$ | $\frac{n!}{(a + j\omega)^{n+1}}$ | $a > 0$ |
| 6 | $\delta(t)$ | 1 | |
| 7 | 1 | $2\pi\delta(\omega)$ | |
| 8 | $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0)$ | |
| 9 | $\cos \omega_0 t$ | $\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$ | |
| 10 | $\sin \omega_0 t$ | $j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$ | |
| 11 | $u(t)$ | $\pi\delta(\omega) + \frac{1}{j\omega}$ | |
| 12 | $\text{sgn } t$ | $\frac{2}{j\omega}$ | |
| 13 | $\cos \omega_0 t u(t)$ | $\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$ | |
| 14 | $\sin \omega_0 t u(t)$ | $\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$ | |
| 15 | $e^{-at} \sin \omega_0 t u(t)$ | $\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$ | $a > 0$ |
| 16 | $e^{-at} \cos \omega_0 t u(t)$ | $\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$ | $a > 0$ |
| 17 | $\text{rect}\left(\frac{t}{\tau}\right)$ | $\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$ | |
| 18 | $\frac{W}{\pi} \text{sinc}(Wt)$ | $\text{rect}\left(\frac{\omega}{2W}\right)$ | |
| 19 | $\Delta\left(\frac{t}{\tau}\right)$ | $\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$ | |
| 20 | $\frac{W}{2\pi} \text{sinc}^2\left(\frac{Wt}{2}\right)$ | $\Delta\left(\frac{\omega}{2W}\right)$ | |
| 21 | $\sum_{n=-\infty}^{\infty} \delta(t - nT)$ | $\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$ | $\omega_0 = \frac{2\pi}{T}$ |
| 22 | $e^{-t^2/2\sigma^2}$ | $\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$ | |

Table 4.2
Fourier Transform Operations

| Operation | $f(t)$ | $F(\omega)$ |
|------------------------------------|----------------------------|--|
| Addition | $f_1(t) + f_2(t)$ | $F_1(\omega) + F_2(\omega)$ |
| Scalar multiplication | $kf(t)$ | $kF(\omega)$ |
| Symmetry | $F(t)$ | $2\pi f(-\omega)$ |
| Scaling (a real) | $f(at)$ | $\frac{1}{ a }F\left(\frac{\omega}{a}\right)$ |
| Time shift | $f(t - t_0)$ | $F(\omega)e^{-j\omega t_0}$ |
| Frequency shift (ω_0 real) | $f(t)e^{j\omega_0 t}$ | $F(\omega - \omega_0)$ |
| Time convolution | $f_1(t) * f_2(t)$ | $F_1(\omega)F_2(\omega)$ |
| Frequency convolution | $f_1(t)f_2(t)$ | $\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$ |
| Time differentiation | $\frac{d^n f}{dt^n}$ | $(j\omega)^n F(\omega)$ |
| Time integration | $\int_{-\infty}^t f(x) dx$ | $\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$ |

Table 6.1
A Short Table of (Unilateral) Laplace Transforms

| | $f(t)$ | $F(s)$ |
|-----|--|--|
| 1 | $\delta(t)$ | 1 |
| 2 | $u(t)$ | $\frac{1}{s}$ |
| 3 | $tu(t)$ | $\frac{1}{s^2}$ |
| 4 | $t^n u(t)$ | $\frac{n!}{s^{n+1}}$ |
| 5 | $e^{\lambda t} u(t)$ | $\frac{1}{s - \lambda}$ |
| 6 | $te^{\lambda t} u(t)$ | $\frac{1}{(s - \lambda)^2}$ |
| 7 | $t^n e^{\lambda t} u(t)$ | $\frac{n!}{(s - \lambda)^{n+1}}$ |
| 8a | $\cos bt u(t)$ | $\frac{s}{s^2 + b^2}$ |
| 8b | $\sin bt u(t)$ | $\frac{b}{s^2 + b^2}$ |
| 9a | $e^{-at} \cos bt u(t)$ | $\frac{s + a}{(s + a)^2 + b^2}$ |
| 9b | $e^{-at} \sin bt u(t)$ | $\frac{b}{(s + a)^2 + b^2}$ |
| 10a | $re^{-at} \cos(bt + \theta) u(t)$ | $\frac{(r \cos \theta)s + (ar \cos \theta - br \sin \theta)}{s^2 + 2as + (a^2 + b^2)}$ |
| 10b | $re^{-at} \cos(bt + \theta) u(t)$ | $\frac{0.5re^{j\theta}}{s + a - jb} + \frac{0.5re^{-j\theta}}{s + a + jb}$ |
| 10c | $re^{-at} \cos(bt + \theta) u(t)$ | $\frac{As + B}{s^2 + 2as + c}$ |
| | $r = \sqrt{\frac{A^2 c + B^2 - 2ABa}{c - a^2}}, \theta = \tan^{-1} \frac{Aa - B}{A\sqrt{c - a^2}}$ | |
| | $b = \sqrt{c - a^2}$ | |
| 10d | $e^{-at} \left[A \cos bt + \frac{B - Aa}{b} \sin bt \right] u(t)$ | $\frac{As + B}{s^2 + 2as + c}$ |
| | $b = \sqrt{c - a^2}$ | |

Table 6.2

The Laplace Transform Properties

| Operation | $f(t)$ | $F(s)$ |
|---------------------------|----------------------------------|--|
| Addition | $f_1(t) + f_2(t)$ | $F_1(s) + F_2(s)$ |
| Scalar multiplication | $kf(t)$ | $kF(s)$ |
| Time differentiation | $\frac{df}{dt}$ | $sF(s) - f(0^-)$ |
| | $\frac{d^2f}{dt^2}$ | $s^2F(s) - sf(0^-) - \dot{f}(0^-)$ |
| | $\frac{d^3f}{dt^3}$ | $s^3F(s) - s^2f(0^-) - sf'(0^-) - \ddot{f}(0^-)$ |
| Time integration | $\int_{0^-}^t f(\tau) d\tau$ | $\frac{1}{s}F(s)$ |
| | $\int_{-\infty}^t f(\tau) d\tau$ | $\frac{1}{s}F(s) + \frac{1}{s} \int_{-\infty}^{0^-} f(t) dt$ |
| Time shift | $f(t - t_0)u(t - t_0)$ | $F(s)e^{-st_0} \quad t_0 \geq 0$ |
| Frequency shift | $f(t)e^{s_0t}$ | $F(s - s_0)$ |
| Frequency differentiation | $-tf(t)$ | $\frac{dF(s)}{ds}$ |
| Frequency integration | $\frac{f(t)}{t}$ | $\int_s^{\infty} F(z) dz$ |
| Scaling | $f(at), a \geq 0$ | $\frac{1}{a}F\left(\frac{s}{a}\right)$ |
| Time convolution | $f_1(t) * f_2(t)$ | $F_1(s)F_2(s)$ |
| Frequency convolution | $f_1(t)f_2(t)$ | $\frac{1}{2\pi j}F_1(s) * F_2(s)$ |
| Initial value | $f(0^+)$ | $\lim_{s \rightarrow \infty} sF(s) \quad (n > m)$ |
| Final value | $f(\infty)$ | $\lim_{s \rightarrow 0} sF(s) \quad (\text{poles of } sF(s) \text{ in LHP})$ |