ELC 3335 – Signals and Systems
Fall 2018
Test 2 – November 13, 2018
Closed Book/Closed Notes/Formula Sheet Provided
1 hour and 15 minutes

1. The exam is closed-book/closed-notes. Only the formula sheet provided with the exam may be used.

2. A calculator may be used to assist with the test. No laptops or tablets are allowed. No cellular phones may be used in any way during the test. Unauthorized electronic device use will result in disqualification.

3. You must circle or box your answers to get full credit.

4. All work and steps toward a solution must be clearly shown to obtain credit.

5. Partial credit may be given provided that the grader can clearly follow your work to the extent that an understanding of the problem is demonstrated.

6. No collaboration is allowed on this examination. Only Dr. Baylis or the exam proctor may be consulted for clarification.

7. You may attach extra sheets to the exam if necessary. Each page should contain your name, the problem number, and the page number for that problem.

Please sign the statement below. YOU MUST SIGN THE STATEMENT OR YOU WILL GET A ZERO FOR THIS EXAMINATION!!

I hereby testify that I have neither provided or received information from unauthorized sources during the test and that this test is the sole product of my effort.

Signed ______________________________________                          Date_____________________
PROBLEM 1 (25 points): Consider the periodic function $f(t)$:

(a) (5 points) Find the value of $a_0$ in the cosine/sine trigonometric Fourier series.

(b) (10 points) Find the value of $a_n$ (in terms of $n$ for $n \geq 1$).

(continued on next page)
(c) (10 points) Find the value of $b_n$ (in terms of $n$ for $n \geq 1$).
**PROBLEM 2 (20 points):** The compact trigonometric Fourier series of a certain periodic signal is given by

\[ f(t) = 1 + 3 \cos \left( t - \frac{\pi}{4} \right) + 5 \cos \left( 3t + \frac{\pi}{2} \right) + 3 \cos \left( 4t - \frac{\pi}{6} \right) + 2 \cos \left( 5t - \frac{\pi}{3} \right) \]

(a) (7 points) Sketch the compact trigonometric Fourier series spectra (that is, provide plots of \( C_n \) and \( \theta_n \) versus \( \omega \)). Clearly label all radian frequencies, amplitudes, and phases of the spectral components on your plots.

(b) (6 points) From the trigonometric spectral plots, plot the **exponential** Fourier series magnitude and phase spectra. Clearly label all radian frequencies, amplitudes, and phases of the spectral components on your plots.
(c) (7 points) By inspection of the plot in part (b), write the exponential trigonometric Fourier series expression for \( f(t) \).
PROBLEM 3 (20 points): Using the definition of the Fourier transform, find the Fourier transform of $f(t)$, where $f(t)$ is shown below and defined as follows:

$$f(t) = \begin{cases} 
2 & -1 \leq t < 0 \\
e^t & 0 \leq t < 2 \\
0 & \text{elsewhere}
\end{cases}$$
**PROBLEM 4 (15 points):** Apply and derive properties of the Fourier transform as described:

(a) (10 points) Use Table 4.1 and Table 4.2 to find the Fourier transform of the following function $f(t)$. Do not use the definition; use an appropriate pair from Table 4.1 and an appropriate property from Table 4.2.

(b) (5 points) Use the Frequency Shift property from Table 4.2 with Euler’s Formula to derive the modulation property, which states that

$$f(t) \cos \omega_0 t \Leftrightarrow \frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$$
PROBLEM 5 (20 points): Determine the Nyquist sampling rate and the Nyquist sampling interval for the signal

\[ f(t) = \text{sinc}(32\pi t) \text{sinc}^2(16\pi t) \]